AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #3 Due June 27, 2008

1. The Runge function is
$$r(x) = \frac{1}{1+x^2}, \quad -5 \le x \le 5.$$
   (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n+1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?
   (b) Repeat part (a) but now use the interpolation points
$$x_j = 5\cos\frac{(2j-1)\pi}{2n+2}, \quad j = 1, \ldots, n+1.$$
   What difference do you observe ?

2. Given the data $(0,3), (1,3), (2,3), (3,6)$, calculate $P_3(x)$, the cubic polynomial interpolating this data (by hand) in three ways:
   (a) Solve the Vandemonde system.
   (b) Use Lagrange Polynomials.
   (c) Find the Newton form, using the divided difference table.

3. For $f(x) = \ln(x+1)$ we are given that
$$f(0) = 0, \ f'(0) = 1, \ f(1) = .6931, \ f'(1) = .5$$
   Calculate an approximation to $f(0.5)$ using cubic Hermite interpolation. Compare the result with $f(0.5) = .4055$.

4. Consider the function S(x) defined as
$$S(x) = \begin{cases} 26 + 26x + 9x^2 + x^3, & -3 \le x \le -1, \\ 24 + 20x + 3x^2 - x^3, & -1 \le x \le 0, \\ 24 + 20x + 3x^2 - 2x^3, & 0 \le x \le 3, \\ -165 + 209x - 60x^2 + 5x^3, & 3 \le x \le 4. \end{cases}$$
   Show that $S(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. (A natural cubic spline is a spline $S(x)$ which satisfies $S''(x_1) = S''(x_N) = 0$) Be sure to state explicitly each of the properties of $S(x)$ which are necessary for this to be true.

5. The following table gives values for a property of titanium as a function of temperature T,

T	*605	*645	685	*725	*765
C(T)	0.622	0.639	0.655	0.668	0.679
T	*795	825	*845	865	*875
C(T)	0.694	0.730	0.812	1.044	1.336
T	*885	*895	905	*915	*925
C(T)	1.881	2.169	2.075	1.598	1.211
T	935	*955	*975	*1015	*1065
C(T)	0.916	0.672	0.615	0.603	0.601

Interpolate these data at the starred points with the cubic spline $S(x)$ using the MATLAB function SPLINE and plot the results. How well does $S(x)$ do at other points ?

6. Ex.3.3, p.110, *Numerical Computing with MATLAB* .
7. Ex.3.16, p.116, *Numerical Computing with MATLAB* .
8. Ex.3.17, p.116, *Numerical Computing with MATLAB* .
9. Ex.5.8, p.161, *Numerical Computing with MATLAB* .
10. Ex.5.12, p.164,*Numerical Computing with MATLAB*
11. For the CENSUSGUI data on p.144 find the cubic polynomial $p_3(s) = \beta_4 s^3 + \beta_3 s^2 + \beta_2 s + \beta_1$, where s is the translated and scaled time variable, which interpolates the data in the sense of least squares by constucting the 11×4 data matrix A and finding the vector $(\beta_1, \beta_2, \beta_3, \beta_4)^T$ in four different ways:
    (a) By using the backslash operator.
    (b) By forming and solving the normal equations. Note the condition number of the matrix $A^T A$.
    (c) By using the QR decomposition.
    (d) By using the Singular-Value Decomposition.

    All of this is quite easy in MATLAB. Compare with the values given by POLYFIT.