

- Write a MATLAB program to evaluate $I = \int_a^b f(x) dx$ using the trapezoidal rule with n subdivisions, calling the result I_n . Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \dots, 512$.

$$(a) \int_0^1 \sqrt{16 + x^2} dx \quad (b) \int_0^1 x^{3/7} dx$$

The exact value of the integral in (a) is $I = 4.04128450518694$.

Analyze empirically the rate of convergence of I_n to I by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \text{ and } p_n = \frac{\log(R_n)}{\log(2)}$$

In part (b) compute the extrapolated approximation to I ,

$$I \approx I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

for $n = 128$.

- Repeat problem 1 using Simpson's rule.
- Apply the corrected trapezoidal rule to the integral in problem 1(a). Compare the results with those of problem 2 for Simpson's rule.
- Use Gauss-Legendre integration with $n = 2, 4, 8$ nodes to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson methods.
- Find approximate values of the integral in problem 1(a) by computing the Romberg integral $I_{32}^{(5)}$ where $I_n^{(0)}$ is the n -panel trapezoid approximation and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_{n/2}^{(k-1)}}{4^k - 1}$$

for n divisible by 2^k .

- Use the MATLAB function QUADL to find approximate values of the integrals 1(a) and 1(b).
- Ex. 6.13 p.181 *Numerical Computing with MATLAB*.
- The 11 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x) dx \approx \sum_{i=0}^{10} w_i f\left(\frac{i}{10}\right)$$

with the w_i determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \dots, x^{10}$.

- (a) Use MATLAB to find the weights w_i .
- (b) Apply the rule to the integrals in 1(a) and 1(b). Note the errors.

9. We wish to estimate the value of

$$I = \int_0^\infty x^{1/2} e^{-x} dx = \frac{1}{2} \sqrt{\pi}$$

- (a) Truncate the integral and use QUAD on the finite part.
 - (b) Try the transformation $x = -\ln t$ on this integral and use QUADL on the new integral. (QUADL will complain but will do it).
 - (c) Use the 2, 4 and 8 point Gauss-Laguerre rules to estimate the integral. compare your results with parts (a) and (b) above.
10. In a standard shell and tube heat exchanger hot vapor condenses on the tube, maintaining a constant temperature T_s . If the input is at temperature T_1 and the output must be at temperature T_2 , then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_p dT}{h(T_s - T)}$$

(All quantities must be in consistent units.) Here T is the temperature in °F.

$T_1 = 60^\circ\text{F}$ is the inlet temperature.

$T_2 = 500^\circ\text{F}$ is the desired outlet temperature.

$T_s = 550^\circ\text{F}$ is the condensate temperature.

m is the fluid flow rate = 22.5 lb/hr.

D is the diameter of the tube = 0.495 in.

c_p is the specific heat of the fluid = $0.251 + 3.46 \times 10^{-5}T - \frac{14,400}{(T+460)^2}$ BTU/(lb°F).

h is the local heat transfer coefficient = $\frac{0.023k}{D} \left(\frac{4m}{\pi D \mu}\right)^{0.8} \left(\frac{\mu c_p}{k}\right)^{0.4}$.

μ is the viscosity of the fluid = $0.0332 \left(\frac{T+460}{460}\right)^{0.935}$ lb/(ft hr).

k is the thermal conductivity of the fluid and has the units BTU/(hr ft°F). k varies with temperature so that

T	0	200	300	400	500
k	.0076	.0130	.0157	.0183	.0209

Use spline interpolation to define k for other values of T and calculate the required length of the heat exchanger.

You will need to use the MATLAB functions SPLINE and QUADL. The answer is about 5.9 feet.