AMSC/CMSC 460 SUMMER 2008

SAMPLE MIDTERM EXAM

1. Assume a decimal (base 10) floating point system having machine precision $\epsilon_{mach} = 5 \times 10^{-6}$ and an exponent range of ± 20. What is the result of each of the following floating-point operations

(a) $1 + 10^{-7}$	(b) $1 + 10^{3}$	(c) $1 + 10^{7}$
(d) $10^{10} + 10^{3}$	(e) $10^{10}/10^{-15}$	(f) $10^{-10} \times 10^{-15}$

2.
(a) Traditional Numerical Analysis doctrine says that one rarely, if ever, computes the inverse of a matrix. Why is this?
(b) Suppose A, B and C are $n \times n$ matrices with B and C nonsingular, and $\mathbf{b} \in \mathbf{R^n}$. Write a MATLAB script which evaluates

$$\mathbf{x} = B^{-1}(2A + I)(C^{-1} + A)\mathbf{b}$$

without computing any matrix inverses. (In MATLAB I =eye(n).)

3. Let

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

(a) Find a permutation matrix P, a lower triangular matrix L and an upper triangular matrix U such that $A = PLU$ with the factorization corresponding to Gauss elimination with partial pivoting.
(b) Use the factorization to solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = (10, 5)^T$.

4. Given the three data points $(-1, 3), (0, 5), (1, 9)$, find the interpolating quadratic:
(a) in the form $ax^2 + bx + c$ by solving a system of linear equations.
(b) in the Lagrange form
(c) in a Newton form.
Show that the three representations give the same polynomial.

5. Find the linear function which best fits the data points of problem 4 in the sense of least squares.

6. A natural cubic spline S is defined by

$$S(x) = \begin{cases} 1 + B(x-1) - D(x-1)^3 & \text{for } 1 \le x \le 2, \\ 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$$

If S interpolates the data $(1, 1), (2, 1)$ and $(3, 0)$, find B, D, b, and d.

7. The (simple) Trapeziodal rule applied to $\int_0^a f(x), dx$ gives the value 5 and the Midpoint rule gives the value 4. What value does Simpson's rule give?

8. Consider the integration rule

$$I = \int_0^{3h} f(x)\,dx \approx Q_h = \frac{9}{4}hf(h) + \frac{3}{4}hf(3h).$$

(a) Find its degree of precision, that is find the largest integer k such that the formula is exact for all polynomials of degree $\leq k$.

(b) Given that the error of the rule is of the form

$$e = I - Q_h = cf^{(n)}(\zeta)h^n$$

for smooth f, where $0 < \zeta < 3h$, find the values of n and c.