Math 462 Dr. Wolfe ASSIGNMENT #2 Due October 15, 2009

1. p. 54, Ex.16, **Strauss**.
2. p. 54, Ex. 18, **Strauss**.
3. p. 67, Ex 9, **Strauss**.
4. p. 79, Ex 1, **Strauss**.
5. Consider the initial value problem

$$u_{tt} = u_{xx},\ t > 0, -\infty < x < \infty, \qquad u(x,0) = 0, \quad u_t(x,0) = g(x)$$

where

$$g(x) = \begin{cases} 1, & |x| \le 2 \\ 0, & |x| > 2 \end{cases}$$

(a) The data g has singularities at $x = \pm 2$. Make a sketch showing the characteristics emanating from $x = \pm 2$. These lines divide up the (x,t) plane, $t \ge 0$ into six regions, $R_1, R_2, \ldots, R_6$.

(b) For (x,t) in each of the regions, show that d'Alembert's formula reduces to the following collection of formulas for the solution (when you label the regions appropriately): $u(x,t) = 0$ in R_1, R_6. $u(x,t) = t$ in R_2. $u(x,t) = 2$ in R_3. $u(x,t) = (x+t+2)/2$ in R_4. $u(x,t) = (t+2-x)/2$ in R_5.

(c) Show that the solution of this IVP can also be represented as

$$u(x,t) = F(x-t) + G(x+t),$$

where

$$F(x) = \begin{cases} 1, & x < -2 \\ -x/2, & -2 \le x \le 2 \\ -1, & x > 2 \end{cases}$$

and

$$G(x) = -F(x).$$