AMSC/CMSC 466 Dr. Wolfe ASSIGNMENT #6 Due December 8, 2004

1.
   (a) Implement the bisection method in MATLAB to find the smallest positive root of
   $$e^{-x} = \sin x \tag{1}$$
   (b) Solve (1) using the secant method. (Use either a calculator or MATLAB.)

2. Write a MATLAB function Newton(f, df, x, tol) to implement Newton's method. You need to supply functions $f(x)$ and $df(x)(f'(x))$. The input x is the initial guess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.
   (a) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten positive solutions of
   $$x = \tan x.$$
   Note: The careful selection of x is critical.
   (c) Try the function on the double root $x = 2$ of
   $$x^3 - x^2 - 8x + 12 = 0.$$
   Use $x = 3$ and $tol = 10^{-6}$. What is the rate of convergence ?
   (d) Newton can be used to find complex roots also. By starting with a non-real initial guess, find the complex roots of
   $$x^3 + 2x - 5 = 0.$$

3. Write down two fixed point procedures for finding a zero of the function $f(x) = 2x^2 + 6e^{-x} - 4$. Check that they converge.

4. Ex. 1, p.106, *Atkinson & Han.*

5. Consider the mapping $g(x) = cx(1 - x)$.
   (a) Show that for $0 \le c \le 4$, g maps $[0, 1]$ into itself.
   (b) Show that if $1 < c < 3$, g has a positive fixed point which is attracting.
   (c) For $c = 3.2$ investigate the dynamics of the iteration $x_{n+1} = g(x_n)$. Certain numbers play a significant role here. Identify them as fixed points of some mapping.

6. Ex. 8, p.107, *Atkinson & Han.*

7. Ex.14, p.108, *Atkinson & Han.*

8. Ex.18, p.109, *Atkinson & Han.*

9. Ex. 3, p.364, *Atkinson & Han.*

10. Consider the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 \\ -1 & 4 & -1 & 0 & -1 \\ 0 & -1 & 4 & -1 & 0 \\ -1 & 0 & -1 & 4 & -1 \\ 0 & -1 & 0 & -1 & 4 \end{pmatrix}$$

and $\mathbf{b} = (-2, -1, 6, 7, 14)'$. Solve the system using

(a) The Cholesky factorization of A (MATLAB: CHOL)

(b) Jacobi iteration.

(c) Gauss-Seidel iteration. (The MATLAB command TRIL might be useful.)