AMSC/CMSC 466 FALL 2004 SAMPLE HOUR EXAM II

1. We wish to fit the data (0,1), (1,3), (2,7), (3,10), (4,20) to a function of the form

$$f(x) = a + bx + ce^x$$

in the sense of least squares. Find an equation for the coefficients a, b and c. Do not do any computations.

2. Let

$$I = \int_1^2 \ln x \, dx = .3862943611$$

Compute approximations to I using
(a) The 4 panel trapezoid rule.
(b) The 4 panel Simpson's rule.
(c) The 4 panel corrected trapezoid rule.
Which method gives the best result ?

3. Suppose we use Simpson's rule with 400 panels to approximate

$$I = \int_0^1 (x^3 + 3x^2 - x) dx.$$

Assuming no roundoff error, what will the result be ? Explain.

4.
(a) Find constants A and B so that the integration rule

$$\int_0^1 f(x) \, dx \approx Af(1/3) + Bf(1)$$

is exact for all first degree polynomials.
(b) Is the rule exact for second degree polynomials?
(c) Let $f(x) = e^x$. Apply the rule to f and compare with the exact value of the integral. Compute the relative error in the approximation.

5. Let $g(x) = \cos x$, $x_0 = \pi/6$, $h = \pi/24$.
(i) For this value of h compute the centered difference approximation to $g'(x_0)$. Compute the percent relative error in this approximation.
(ii) For this value of h compute the centered difference approximation to $g''(x_0)$. Compute the percent relative error in this approximation.

6. Let $f(x) = x^3 + 3x - 6$.
(a) Prove that there is a number α with $1 < \alpha < 2$ such that $f(\alpha) = 0$.
(b) Use the bisection method with the initial interval $[1, 2]$ to approximate α with error less than $\frac{1}{16}$.
(c) Let $x_0 = 1$, $x_1 = 2$. Use the secant method to compute a new approximation to α, x_2.
(d) Let $x_0 = 1.5$. Use Newton's method to approximate α. Use the method until successive iterates obtained by your calculator are identical.