AMSC/CMSC 466 Dr. Wolfe ASSIGNMENT #1 Due February 15, 2010

1. What do each of these MATLAB programs do? What are the last two values of x printed? What is the final value of n? Do not hand in any output. Just explain the results.

$$x = 1;\ n = 0,\ \text{while } 1 + x > 1, x = x/2,\ n = n + 1,\ \text{pause}(.02),\ \text{end}$$

$$x = 1;\ n = 0,\ \text{while } x + x > x, x = 2 * x,\ n = n + 1,\ \text{pause}(.02),\ \text{end}$$

$$x = 1;\ n = 0,\ \text{while } x + x > x, x = x/2,\ n = n + 1,\ \text{pause}(.02),\ \text{end}$$

2. Using four digit arithmetic, add the following numbers, first in ascending order (from smallest to largest) and then in descending order. In doing so round off the partial sums to four significant figures. Compare your results with the correct sum $x = 0.107101023$ E5. ($En = 10^n$.)

0.1580 E0	0.4288 E1	0.7767 E3
0.2653 E0	0.6266 E2	0.7889 E3
0.2581 E1	0.7555 E2	0.8999 E4

3. In IEEE *single precision floating-point arithmetic* a non-zero machine number x is represented as

$$x = \pm 1.a_1 a_2 \ldots a_{23} \cdot 2^e$$

where $a_i = 0$ or 1 and $-126 \le e \le 127$. Find the largest integer n such that $n!$ can be exactly represented as a machine number.

4. Suppose a computer carries three decimal digits and rounds. If x and y are machine numbers, define the machine version of addition $x \oplus y$ to be the result of adding x and y and rounding to three digits. For example

$$49.3 \oplus 57.4 = 107.$$

Define machine multiplication $x \otimes y$ similarly. For example,

$$1.23 \otimes 4.86 = 5.98.$$

Constuct examples to show that, in general, the following statements are <u>not</u> true:
(a) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$,
(b) $(x \otimes y) \otimes z = x \otimes (y \otimes z)$,
(c) $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$.

5. Ex.1, p.62, *Atkinson & Han.*

6. Let

$$f(x) = \frac{\ln(1 + x) - \sin x - \cos x + 1}{x^3}.$$

Use MATLAB to compute $f(x)$ for $x = 10^{-m},\ m = 1, 2, \cdots, 12$. According to the theory what is $\lim_{x\to 0} f(x)$? For x near zero what is a better way to compute $f(x)$? (Hint: Use Taylor's theorem on the numerator.)

7. Write a program to compute the first 60 terms in the sequence given by the difference equation

$$x_{k+1} = 2.25x_k - 0.5x_{k-1} \tag{1}$$

with the starting values

$$x_1 = \frac{1}{3} \text{ and } x_2 = \frac{1}{12}.$$

Make a plot of $\log_2 x_k$ (log2 in MATLAB) as a function of k. The exact solution of (1) is given by

$$x_k = \frac{4^{1-k}}{3}$$

which decreases monotonically as k increases. Does your graph confirm this theoretically expected behavior? Can you explain the results? (**Hint:** Find the general solution of (1). To do this, look for solutions of the form $x_k = \alpha^k$ and observe that (1) is linear.)

8. The power series for $\sin x$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots.$$

This MATLAB function uses the series to compute $\sin x$.

```
function s = powersin (x)
% POWERSIN.        Power series for sin(x).
% POWERSIN(x)      tries to compute sin(x)
% from a power series.
s = 0;
t = x;
n = 1;
while s + t ~= s;
                   s = s + t;
                   t = -x. ^ 2/((n + 1) * (n + 2)). * t;
                   n = n + 2;
end
```

What causes the "while" loop to terminate? Answer the following questions for $x = \pi/2, 11\pi/2, 21\pi/2$, and $31\pi/2$.

How accurate is the computed result?

How many terms are required?

What is the largest term in the series?

What do you conclude about the use of floating-point arithmetic and power series to evaluate functions?