ENEE 241/MATH 242   Dr. Wolfe   ASSIGNMENT #1   Due February 10/11, 2004

Readings: *Linz & Wang* , Chapter 2.

1. Run the following MATLAB scripts and explain the output.

$x = 1;$	$x = 1;$	$x = 1;$
while $x + 1 > 1$	while $x > 0$	while x <Inf
$y = x;$	$y = x;$	$y = x;$
$x = x/2;$	$x = x/2;$	$x = 2 * x;$
end	end	end
y	y	y
$p = \log(y)/\log(2)$	$p = \log(y)/\log(2)$	$p = \log(y)/\log(2)$

   Assume $\beta = 2$. What can you infer about the machine constants t, U and L ?

2. Let
$$f(x) = \frac{e^x - 1 - x}{x^2}.$$
   Use MATLAB to compute $f(x)$ for $x = 10^{-m},\ m = 1, 2, \cdots, 10$. Use L'hôpital's rule to compute $\lim_{x \to 0} f(x)$. For x near zero what is a better way to compute $f(x)$? (Hint: Use Taylor's theorem on the numerator.)

3. Problem 6, p.30 *Linz & Wang* .

4. Sometimes the loss of significance error can be avoided by rerranging terms in the function using a known identity from trigonometry or algebra. Find an equivalent formula for the following functions that avoids a loss of significance.
   (a) $\ln(x + 1) - \ln(x)$ for large x.
   (b) $\sqrt{x^2 + 1} - x$ for large x.
   (c) $\cos^2(x) - \sin^2(x)$ for $x \approx \pi/4$.
   (d) $\sqrt{\frac{1+\cos(x)}{2}}$ for $x \approx \pi$.

5. The following algorithm estimates the unit roundoff u by a computable quantity U:
$$A = 4/3$$
$$B = A - 1$$
$$C = B + B + B$$
$$U = |C - 1|$$
   (a) What does the above algorithm yield for U in six-digit *rounded* arithmetic ?
   (b) What does it yield for U in six-digit *chopped* arithmetic ?
   (c) What are the exact values for u in the arithmetics of (a) and (b) ?
   (d) Use the algorithm on MATLAB and your calculator. What do you get ?

6. For $\alpha = 0.8717$ and $\beta = 0.8719$ calculate the midpoint m of the interval $[\alpha, \beta]$ by using the formula $m = (\alpha + \beta)/2$. First use four-digit decimal chopped arithmetic,

then four digit rounded arithmetic. How reasonable are the answers ? Find another formula for the midpoint and use four-digit decimal (rounded or chopped) arithmetic to calculate the midpoint of $[0.8717, 08719]$. Is your formula better or worse ?

7. Use three-digit arithmetic with rounding to compute the following sums (sum in the given order).

$$\text{(a)} \sum_{k=1}^{6} \frac{1}{3^k} \qquad\qquad \text{(b)} \sum_{k=1}^{6} \frac{1}{3^{7-k}}$$

   Also, compare the answers with the exact sum. Which is better ?

8. Suppose N is an integer. In exact arithmetic, adding $1/N$ to itself N times yields an answer exactly equal to 1. In finite precision arithmetic this is not necessarily the case.
   Try out the following MATLAB script. Start by typing "format long" then let (i) $N = 100,000 (= 10^5)$ and (ii) $N = 131,072 (= 2^{17})$. Compute the error relative to the exact answer $(= 1)$ in each case. Explain any differences between the two cses.

$$y = 0;$$
$$\text{for } i = 1 : N$$
$$y = y + (1/N);$$
$$\text{end}$$
$$y$$