ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #10 Due May 11/12, 2004

Readings: *Linz & Wang* , Section 10.2, 10.4, 10.5.

1. (MATLAB) Compute numerical approximations to the solution of the initial value problem
$$\frac{dy}{dx} = 2y + x, \quad y(0) = 1$$
using the Euler, Improved Euler and Runge-Kutta methods. For Euler and Improved Euler take $h = 0.1\,, 0.05,\ 0.025$. For Runge-Kutta take $h = 0.2,\ 0.1$. Print out your results at $x = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0$. Compute the actual errors by comparing your results with the exact solution $y = 1.25e^{2x} - 0.5x - 0.25$. Discuss your results.

2. The following differential equations describe the motion of a body in orbit about two much heavier bodies. An example would be a spacecraft in an earth-moon orbit. We use a rotating coordinate system. The three bodies determine a plane in space and a two-dimensional cartesian coordinate system in this plane. The origin is at the center of mass of the two heavy bodies, the x axis is the line through these two bodies, and the distance between them is taken as the unit of length. Thus if μ is the ratio of the mass of the moon to that of the earth, then the moon and the earth are located at coordinates $(1-\mu, 0)$ and $(-\mu, 0)$ respectively, and the coordinate system moves as the moon revolves about the earth. The third body, the spacecraft, is assumed to have a mass which is negligible compared to the other two, and its position as a function of time is $(x(t), y(t))$. The equations are derived from Newton's law of motion and the inverse square law of gravitation. The first derivatives in the equation come from the rotating coordinate system.

$$\begin{aligned} x'' &= 2y' + x - \frac{\tilde{\mu}(x+\mu)}{r_1^3} - \frac{\mu(x-\tilde{\mu})}{r_2^3} \\ y'' &= -2x' + y - \frac{\tilde{\mu}y}{r_1^3} - \frac{\mu y}{r_2^3}, \end{aligned}$$

with
$$\mu = \frac{1}{82.45},\ \tilde{\mu} = 1 - \mu,\ r_1^2 = (x+\mu)^2 + y^2,\ r_2^2 = (x-\tilde{\mu})^2 + y^2.$$
It is known that the initial conditions
$$x(0) = 1.2,\ x'(0) = 0,\ y(0) = 0,\ y'(0) = -1.04935751$$
lead to a solution which is periodic with period $T = 6.19216933$. This means that the spacecraft starts on the far side of the moon with an altitude of about 0.2 times the earth-moon distance and a certain initial velocity. The resulting orbit brings the spacecraft in close to the earth, out in a big loop on the opposite side of the earth from the moon, back in close to the earth again, and finally back to its original position on the far side of the moon with velocity the same as the initial velocity. Use the MATLAB code **ode45** to compute the solution with the given initial conditions. Plot the orbit in the xy plane. You should see the closed loop. You may need to tighten the tolerence parameters somewhat. Use values of about 10^{-6}. The only output we want is the picture.