ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #3 Due February 24/25, 2004

Readings: *Linz & Wang* , Section 3.2, 3.4, 3.5 up to, but not including, Theorem 3.1, 8.4.

1. The Hilbert matrix of order n, H_n is defined by

$$(H_n)_{i,j} = \frac{1}{i+j-1}, \; i = 1, \ldots, n, \quad j = 1, \ldots, n.$$

   H_n is nonsingular. However, as n increases, the condition number of H_n increases rapidly. H_n is a library function in MATLAB, hilb(n). Let $n = 10, \mathbf{x} = \text{ones}(10, 1)$ and $\mathbf{b} = H_{10}\mathbf{x}$. Now use the backslash operator to solve the system $H_n\mathbf{x} = \mathbf{b}$, obtaining $\mathbf{x}^*$. Since we know $\mathbf{x}$ exactly, we can compute $\mathbf{e} = \mathbf{x} - \mathbf{x}^*$, the error, and $\mathbf{r} = \mathbf{b} - H_{10}\mathbf{x}^*$, the residual. Compute these quantities and also cond(H_n) (a MATLAB function). Show that the two basic principles of solving linear systems by G.E./P.P. in floating point arithmetic hold. How many correct digits does $\mathbf{x}^*$ have ? Repeat with $n = 11, 12, \ldots$. Stop when some component of $\mathbf{x}^*$ has no correct digits.

2. Problem 3, p.41 *Linz & Wang* .

3. Problem 5, p.41 *Linz & Wang* .

4. Problem 4, p.55 *Linz & Wang* .

5. Construct a tridiagonal solver along the lines outlined on p.233 of *Linz & Wang* . Your solver should be a function M-file with four vectors as input and the solution as the output. Test your solver on the problem $A\mathbf{x} = \mathbf{b}$ where A is the 9×9 tridiagonal matrix with 2's on the main diagonal and -1's on the super-and subdiagonals and $\mathbf{b}(j) = .01$, $j = 1, \ldots, 9$. As a check, the answer should be $\mathbf{x}(j) = \frac{1}{2}(\frac{j}{10})(1 - \frac{j}{10})$.