

Readings: *Linz & Wang*, Sections 4.1, 4.2.

1. Write a MATLAB script which takes as input n and plots both $\sin(x)$ and

$$S_n(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

over the interval $[0, 2\pi]$ (on the same graph). Try your script for several values of n and describe the results.

2. Problem 9, p.61 *Linz & Wang*.
3. Problem 1, p.67 *Linz & Wang*. Write a MATLAB script which efficiently generates the matrix for a given n and computes its condition number.
4. Problem 8, p.68 *Linz & Wang*.
5. Consider the function $f(x) = \sin x$ on the interval $[0, \pi]$. Use the error bound stated in class to determine a step size h so that the error in linear interpolation is $< 5 \times 10^{-7}$.
6. The Runge function is

$$r(x) = \frac{1}{1+x^2}, \quad -5 \leq x \leq 5.$$

- (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n+1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better? Where is it getting worse?
- (b) Repeat part (a) but now use the interpolation points

$$x_j = 5 \cos \frac{(2j-1)\pi}{2n+2}, \quad j = 1, \dots, n+1.$$

What difference do you observe?