

Readings: *Linz & Wang*, Sections 3.3, 5.1.

1. Consider the function $S(x)$ defined as

$$S(x) = \begin{cases} 28 + 25x + 9x^2 + x^3, & -3 \leq x \leq -1, \\ 26 + 19x + 3x^2 - x^3, & -1 \leq x \leq 0, \\ 26 + 19x + 3x^2 - 2x^3, & 0 \leq x \leq 3, \\ -163 + 208x - 60x^2 + 5x^3, & 3 \leq x \leq 4. \end{cases}$$

Show that $S(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. (A natural cubic spline is a spline $S(x)$ which satisfies $S''(x_1) = S''(x_N) = 0$) Be sure to state explicitly each of the properties of $S(x)$ which are necessary for this to be true.

2. Problem 3, p.44 *Linz & Wang*.
3. Problem 6, p.45 *Linz & Wang*.
4. (MATLAB) Observed values for the thrust (T) versus time (t) curve of a model rocket are

t	0.00	0.05	0.10	0.15	0.20
T	0.0	1.0	5.0	15.0	33.5
t	0.30	0.40	0.50	0.60	0.70
T	33.0	16.5	16.0	16.0	16.0
t	0.80	0.85	0.90	0.95	1.00
T	16.0	16.0	6.0	2.0	0.0

- (a) Use the MATLAB functions POLYFIT and POLYVAL to find and plot the 14th degree polynomial interpolating this data.
 - (b) Use the MATLAB function SPLINE to find and plot the cubic spline interpolating the data.
 - (c) Which function do you think does a better job of interpolating the data? Why? Suppose we also observe $T(0.25) = 38.0$, $T(0.65) = 16.0$. What values do the interpolation functions give in each case? Compare the results with the observed values.
5. Find the best least squares fit by a linear function $y = \beta_0 + \beta_1x$ to the data points $(-2, 1), (-1, 2), (0, 3), (1, 3), (2, 4)$. Do it by hand and use MATLAB as a check. Plot your linear function along with the data points.
 6. The vapor pressure P of water (in bars) as a function of temperature T ($^{\circ}C$) is

T	0	10	20	30
P(T)	.006107	.012277	.023378	.042433
T	40	50	60	70
P(T)	.073774	.12338	.19924	.31166
T	80	90	100	110
P(T)	.47364	.70112	1.01325	1.22341

We wish to fit this data to a quadratic polynomial $P = \beta_0 + \beta_1 T + \beta_2 T^2$ in the sense of least squares. The MATLAB commands POLYFIT and POLYVAL give you exactly what you need. Plot P along with the points.