ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #6 Due March 30/31, 2004

Readings: *Linz & Wang* , Sections 6.2, 6.3.

This assignment is concerned with the computation of the integral

$$I = \int_0^1 \frac{4}{1+x^2}\, dx = \pi$$

1. Using MATLAB or your calculator use the following methods to compute approximations to I. In each case give the error.
   (a) The simple (1 panel) trapezoid rule.
   (b) The 2 panel trapezoid rule.
   (c) The 4 panel trapezoid rule.
   (d) The simple (2 panel) Simpson's rule.
   (e) The 4 panel Simpsons rule.
   (f) The simple (1 panel) midpoint rule.
   (g) The compound 2 panel midpoint rule.
   (h) The compound 4 panel midpoint rule.
   (i) The three-eighths rule.
   (j) The simple (1 panel) corrected trapeziod rule.
   (k) The 2 panel corrected trapezoid rule.
   (l) The 4 panel corrected trapezoid rule.

2. Download the scripts trap.m and simp.m from the class website. Use them with $n = 8, 16, 32, 64, 128, 256$ to compute approximations to I. In each case compute the error. What do you observe?

3. (a) Find constants α_1 and α_2 such that the integration rule

$$I = \int_0^h f(x)\, dx \approx \alpha_1 f(h/4) + \alpha_2 f(3h/4) = Q$$

   is exact for all linear polynomials.
   (b) Is the above rule exact for quadratics ?
   (c) Given that the error of the rule is of the form

$$e = I - Q = c f^{(d)}(\zeta) h^{d+1}$$

   for smooth f, where $0 < \zeta < h$, find the integer d and the constant c.
   (d) Use the rule to compute an approximation to I. Compute the error. How does it compare with the other methods?