**Readings:** Linz & Wang, Sections 6.3, 6.5.

The first three problems are concerned with the computation of the integral

$$I = \int_0^1 \frac{4}{1+x^2} \, dx = \pi$$

- 1. Use the MATLAB function QUADL on I. Use several different values of TOL.
- 2. Compute the Gauss-Legendre approximations of I with n = 2, 4, 6, 8. I have supplied an m-file with the weights and nodes. Remember, you have to make a change of variable to transform the integal to one whose limits of integration are -1 and 1.
- 3. The 11 point Newton-Cotes integration rule on [0, 1] is

$$\int_{0}^{1} f(x) \, dx \approx \sum_{i=0}^{10} w_i f(\frac{i}{10})$$

with the  $w_i$  determined by requiring that the rule be exact for  $f(x) = 1, x, x^2, \dots x^{10}$ .

- (a) Use MATLAB to find the weights  $w_i$ .
- (b) Apply the rule to I. Note the error.

Problems 4 & 5 refer to the integral

$$J = \int_2^3 \frac{dx}{5-x}$$

- 4. Determine N so that the N-panel trapezoidal rule can be used to compute J with an accuracy of  $5 \times 10^{-9}$ .
- 5. Determine N so that the N-panel Simpson's rule can be used to compute J with an accuracy of  $5 \times 10^{-9}$ .
- 6. In a standard shell and tube heat exchanger hot vapor condenses on the tube, maintaining a constant temperature  $T_s$ . If the input is at temperature  $T_1$  and the output must be at temperature  $T_2$ , then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_\rho dT}{h(T_s - T)}.$$

(All quantities must be in consistent units.) Here T is the temperature in °F.  $T_1 = 0$ °F is the inlet temperature.  $T_2 = 180$ °F is the desired outlet temperature.  $T_s = 250^{\circ}$ F is the condensate temperature.

m is the fluid flow rate = 45,000 lb/hr.

D is the diameter of the tube = 1.032 in.

 $c_{\rho}$  is the specific heat of the fluid = (0.53 + 0.00065T) BTU/(lb°F). h is the local heat transfer coefficient =  $\frac{0.023k}{D} (\frac{4m}{\pi D\mu})^{0.8} (\frac{\mu c_{\rho}}{k})^{0.4}$ .

k is the thermal conductivity of the fluid =  $0.153 \,\mathrm{BTU}/(\mathrm{hr} \,\mathrm{ft}^\circ \mathrm{F})$ .

 $\mu$  is the viscosity of the fluid and has units lb/(ft hr).  $\mu$  varies with temperature so that

Т	0	50	100	150	200
$\mu$	242	82.1	30.5	12.6	5.57

Use spline interpolation to define  $\mu$  for other values of T and calculate the required length of the heat exchanger.

You will need to use the MATLAB functions SPLINE and QUADL . The answer is about 158.7 ft.