ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #8 Due April 20/21, 2004

Readings: *Linz & Wang* , Sections 7.1,7.2,7.3,7.5 (through the bottom of p.191).

1.
   (a) Implement the bisection method in MATLAB to find the smallest positve root of

$$e^{-x} = \sin x \tag{1}$$

   (b) Solve (1) using the secant method. (Use either a calculator or MATLAB .)
   (c) Solve (1) using the MATLAB code FZERO.

2. Write a MATLAB function Newton(f, df, x, tol) to implement Newton's method. You need to write function m-files $f(x)$ and $df(x)(f'(x))$. The input x is the initial quess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.
   (1) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten positive solutions of

$$x = \tan x.$$

   (Zero is not a positive number.) Note: The careful selection of x is critical.
   (c) Try the function on the double root $x = 1$ of

$$x^3 + 3x^2 - 9x + 5 = 0.$$

   Use $x = 2$ and $tol = 10^{-5}$. What is the rate of convergence ?

3. Let $f(x) = \arctan(x)$. Then f has a unique zero: $x = 0$.
   (a) Write out the formula for the Newton iterations for finding the zero of f.
   (b) What happens to the iterations if we start with $x_0 = 1$?
   (c) What happens to the iterations if we start with $x_0 = 2$? Explain this result by drawing a picture of what is happening.

4. Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

(*a*)
$$x_{n+1} = -16 + 6x_n + \frac{12}{x_n} \qquad \alpha = 2$$

(*b*)
$$x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2} \qquad \alpha = 3^{1/3}$$

(c) $$x_{n+1} = \frac{12}{1+x_n} \qquad \alpha = 3$$

5. Let $g(x) = 2e^{-x}$. If we draw the graph of g we see that it has a fixed point α where $\alpha \approx .85$. If we take $x_0 = .8$ and use the fixed point iterations $x_{n+1} = 2e^{-x_n}$ we find it takes about 200 iterations until two succesive iterates are identical in MATLAB , in 'format long'. However, investigate what happens when we use the Aitken extrapolation scheme
$$y = g(x), \quad z = g(y), \quad x = z - \frac{(z-y)^2}{((z-y)-(y-x))}.$$

6 A long conducting rod of diameter D meters and electrical resistance R per unit length is in a large enclosure whose walls (far away from the rod) are kept at temperature $T_s^\circ C$. Air flows past the rod at temperature T_∞. If an electrical current I passes through the rod, the temperature of the rod eventually stabilizes to T, where T satisfies
$$Q(T) = \pi Dh(T - T_\infty) + \pi D\epsilon\sigma(T^4 - T_s^4) - I^2R = 0$$
where

$$\begin{aligned}
\sigma &= \text{Stefan-Boltzman constant} = 5.67 \cdot 10^{-8}\text{Watts/meter}^2\text{Kelvin}^4,\\
\epsilon &= \text{rod surface emissivity} = 0.8,\\
h &= \text{heat transfer coefficient of air flow} = 20\text{Watts/meter}^2\text{Kelvin},\\
T_\infty &= T_s = 25^\circ C,\\
D &= 0.1 \text{ meter},\\
I^2R &= 100.
\end{aligned}$$

Use FZERO to find the steady state temperature T of the rod.