ENEE241/MATH242 Dr. Wolfe FIRST HOUR EXAM March 3, 2000

Instructions: Number your answer sheets from 1 to 3. Fill out all information at the top of each sheet. Do problem n on sheet n, $n = 1, 2, 3$. SHOW ALL WORK.

1. (25 points)

(a) (15 points) Assume a decimal (base 10) floating point system having machine epsilon $\epsilon_{mach} = 5 \times 10^{-6}$ and an exponent range of ± 20. What is the result of each of the following floating-point operations ?

(i) $1 + 10^{-7}$	(ii) $1 + 10^{3}$	(iii) $1 + 10^{7}$
(iv) $10^{10} + 10^{3}$	(v) $10^{10}/10^{-15}$	(vi) $10^{-10} \times 10^{-15}$

(b) (10 points) How can values of $f(x) = \sqrt{x+4} - 2$ be computed accurately (avoiding loss of significance) when x is small ?

2. (40 points) Let $f(x) = x^3 + 2x - 7$.

(a) (3 points) Show there is a number α with $1 < \alpha < 2$ such that $f(\alpha) = 0$. ("This is clear from looking at the graph of f on my graphing calculator" is not an acceptable answer.)

(b) (10 points) Use the bisection method with the initial interval $[1, 2]$ to approximate α with error less than $\frac{1}{16}$.

(c) (10 points) Let $x_0 = 1.5$. Use Newton's method twice to compute new approximations to α, x_1 and x_2.

(d) (10 points) Let $x_0 = 1, x_1 = 2$. Use the secant method twice to compute new approximations to α, x_2 and x_3.

(e) (7 points) Will the iteration scheme

$$x_{n+1} = \frac{1}{2}(7 - x_n^3), \quad x_0 = 1.5$$

converge to α ? Explain.

3. (35 points)

(a) (25 points) Let

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{pmatrix}$$

(i) (15 points) Write $A = LU$ where L is lower triangular and U is upper triangular.

(ii) (10 points) Use the decompision of part (i) to solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = (1, 0, 1)'$ by forward elimination and back substitution.

(b) (10 points) Suppose you enter the vectors $a =$ [1 2 3] and $b =$ [4 5 6] into MATLAB. What will be the result if you enter the following commands ?

(i) $a * b$	(ii) $a. * b$	(iii) $a * b'$