Dr. Wolfe MATH 461 MATLAB PROJECT #3 Due November 20, 2002

Eigenvalues and eigenvectors of square matrices can be found with the command **eig**. If A is a square matrix $d = \mathbf{eig}(A)$ produces a vector containing the eigenvalues of A and $[V, D] = \mathbf{eig}(A)$ produces a diagonal matrix D of eigenvalues and a matrix V whose columns are the corresponding eigenvectors so that $A * V = V * D$.

1. Ex. 33, p.326, *Lay.* Call the matrix A. We will do the problem in two ways:
   (a) Do $d = \mathbf{eig}(A)$. Then find the eigenvectors by row reduction, i.e. do $R = \mathbf{rref}(A - d(1) * \mathbf{eye}(4))$, etc. If you have vectors $p1, p2, p3, p4$, to form them into a matrix P, write $P = [p1\ \ p2\ \ p3\ \ p4]$. Then check that $P * \mathbf{diag}(d) * \mathbf{inv}(P) = A$.
   (b) Do $[V, D] = \mathbf{eig}(A)$ and check that $A = V * D * \mathbf{inv}(V)$. Note that V and P from part (a) may be quite different.

2. Ex. 15, p.341, *Lay.* Call the matrix B. Do $[V, D] = \mathbf{eig}(B)$. Then take

$$P = [\mathbf{real}((V(:, 1))\ \ \mathbf{imag}((V(:, 1))]$$

   and check that $\mathbf{inv}(P) * B * P$ has the correct form.

3.
   (a) Find the general solution of $x' = Ax$, where

$$A = \begin{pmatrix} 3 & -1 & -6 & 0 \\ 0 & 4 & 2 & 6 \\ 3 & -3 & -7 & -3 \\ -5 & 3 & 10 & 2 \end{pmatrix}$$

   (b) Find the solution of the initial value problem $x' = Ax,\ x(0) = x_0$ where $x_0 = (1, 2, -1, 3)^T$.

   Note: To get the k^{th} column vector of a matrix V write $V(:, k)$.

4. The solution of $x' = Ax,\ x(0) = (1, 0)^T$ where A is the matrix of Ex. 14, p.361 is given by

$$x_1 = \cos(2t) - \sin(2t), \quad x_2 = -4\sin(2t).$$

   We wish to see what the trajectory of this solution looks like. So do

$$t = 0 : .01 : \mathrm{pi}; x1 = \cos(2 * t) - \sin(2 * t); x2 = -4 * \sin(2 * t); \mathbf{plot}(x1, x2)$$

   Use the command **print** to print out the resulting graph. Remember that you cannot save the graph in your diary.

5. Let A and $\mathbf{x_0}$ be as in Ex. 10, p.369. Apply the power method to find the largest eigenvalue of A. Work interactively and stop when $A\mathbf{x} = \lambda\mathbf{x}$ in the short format. To normalize your vectors do $x = y/\mathbf{max}(\mathbf{abs}(y))$.