Math 140 **Final Examination** Fall, 2002

Instructions: Answer each of the 10 numbered problems on a separate answer sheet. Each answer sheet must have your name, your TA's name, and the problem number (=page number). Show all your work for each problem clearly on the answer sheet for that problem. You must show enough written work to **justify your answers.**

NO CALCULATORS

1. (8 points each) In each of the following, determine whether the limit exists as a real number, as ∞ or $-\infty$, or fails to exist. If the limit exists, evaluate it.

   a) $\displaystyle\lim_{x\to\infty}\frac{\sin 5x}{3x}$ b) $\displaystyle\lim_{y\to 1^-}\frac{\sqrt{1-y^2}}{y-1}$ c) $\displaystyle\lim_{x\to 0}\frac{5e^{3x}-5}{2x}$

2. (8 points each) Compute the following derivatives (you do NOT need to simplify your answers):

   a) $\displaystyle\frac{d}{dx}\left(xe^{(x^2)}\right)$ b) $\displaystyle\frac{d}{dt}\left(\int_{4t}^{t^2} x\ln(1+\sqrt{x})\,dx\right)$ c) $\displaystyle\frac{d}{dt}\left(\ln(\tan t+\sec t)\right)$

3. (20 points) An isosceles triangle has base 6 and height 10. Find the maximum possible area A of a rectangle that can be placed inside the triangle with one side on the base of the triangle. Explain why your answer gives the maximum area as opposed to the minimum.

4. (10 points each)
   a) Is there a solution to the equation $xe^x = 3$ in the closed interval $[2, 3]$? Explain your reasoning.
   b) Find an equation (in any form) of the line L that is tangent to the graph of $xe^y = ye^x - y^2$ at the point $(0, 1)$.

5. (20 points) A ball is shot upward from the ground with initial velocity k feet/second. Its velocity after t seconds is $v(t) = k - 32t$ feet/second. If the ball rises 2000 feet in the first 2 seconds, what is its initial velocity?

please turn over —->

6. (5 points each) Let $f(x) = \dfrac{1}{x^2-4}$, so $f'(x) = \dfrac{-2x}{(x^2-4)^2}$ and $f''(x) = \dfrac{6x^2+8}{(x^2-4)^3}$

   a) Find all relative maximum values and all relative minimum values of f (if any).

   b) Find the intervals on which the graph of f is concave upward (if any).

   c) Find all inflection points on the graph of f (if any).

   d) Find all vertical and horizontal asymptotes of the graph of f (if any).

7. (15 points) Suppose that the surface of a drop of dew is a perfect sphere and that during the night, the volume of the dew drop increases at a rate that is a constant multiple of the surface area of the dew drop. Compute $\dfrac{d^2r}{dt^2}$, the second derivative of the radius of the dew drop with respect to time t. [Note: The volume of a sphere of radius r is $\dfrac{4}{3}\pi r^3$ and the surface area is $4\pi r^2$.]

8. (9 points each) Evaluate the following integrals:

   a) $\displaystyle\int_2^3 (x^3+x)\sqrt{x^2+1}\,dx$ b) $\displaystyle\int_{1/2}^3 \frac{1}{2u+1}\,du$ c) $\displaystyle\int e^{-2y}(\pi+4e^{2y})\,dy$

9. (15 points) Find the area A of the bounded region to the right of the y-axis and between the graphs of $y = 3x$ and $y = 4 - x^2$.

10. (15 points) Identify the conic section with equation $4x^2 - 9y^2 + 8x + 18y + 4 = 0$. Then determine its foci, vertices, center, and asymptotes (if any), and sketch the graph of the conic section.

END OF EXAM – GOOD LUCK!