ENEE241/MATH242 Dr. Wolfe SAMPLE SECOND HOUR EXAM

1. We wish to solve the system

$$x^2 + 4y^2 = 4, \quad y = x^2$$

by using Newton's method for systems. Let $(x_0, y_0) = (1, 1)$. What is (x_1, y_1) ? Do you think that (x_1, y_1) is closer to a root than (x_0, y_0) ? Do not reduce the system to a single equation.

2. Let $f(x) = \cos \frac{(x-1)\pi}{3}$. (Remember, angles are measured in radians.)
   (i) Find the quadratic polynomial $p_2(x)$ which interpolates $f(x)$ at $x = 0$, $x = 1$ and $x = 2$. Give the Lagrange form, a Newton form, and the standard form of $p_2(x)$.
   (ii) Compute $p_2(1/2)$. Compare the actual value of $f(1/2) - p_2(1/2)$ with the theoretical error bound for quadratic interpolation.

3. Let

$$s(x) = \begin{cases} x^3 - 3x^2 + 2x + 1, & 1 \le x \le 2 \\ -x^3 + 9x^2 - 22x + 17, & 2 \le x \le 3 \end{cases}$$

Is $s(x)$ a cubic spline ? Is it a natural cubic spline ?

4. Find the best least squares fit by a linear function $y = \beta_0 + \beta_1 x$ to the data points $(-1, 0), (0, 2), (1, 5), (2, 8)$. Plot your linear function along with the data points in the xy plane.

5. Consider the linear system

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 22 \\ -x_1 + 8x_2 + 2x_3 &= 30 \\ x_1 - 3x_2 + 12x_3 &= 41 \end{aligned}$$

by Jacobi iterations. If our first quess is $(x_1, x_2, x_3) = (1, 2, 3)$ what is the next iterate ?