

Readings: *Chapra & Canale.*, Sections 5.2,5.3,6.1,6.2,6.3 .

1.

- (a) Implement the bisection method in MATLAB to find the smallest positive root of

$$e^{-x} = \sin x \quad (1)$$

- (b) Solve (1) using the secant method. (Use either a calculator or MATLAB .)
 (c) Solve (1) using the MATLAB code FZERO.

2. Write a MATLAB function `Newton(f, df, x, tol)` to implement Newton's method. You need to write function m-files $f(x)$ and $df(x)(f'(x))$. The input x is the initial guess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.

- (1) Try your function to solve equation (1). Print out the iterates and the function values.
 (b) Use your function to find the first ten positive solutions of

$$x = \tan x.$$

(Zero is not a positive number.) Note: The careful selection of x is critical.

- (c) Try the function on the double root $x = 1$ of

$$x^3 + 3x^2 - 9x + 5 = 0.$$

Use $x = 2$ and $tol = 10^{-5}$. What is the rate of convergence ?

3. Problem 5.8, p.138 *Chapra & Canale.*

4. Let $f(x) = \arctan x$. The f has a unique zero; $x = 0$.

- (a) Write out the formula for the Newton-Raphson iterations for finding the root of f .
 (b) What happens to the iterations if we start with $x_0 = 1$?
 (c) What happens to the iterations if we start with $x_0 = 2$? Explain this result by drawing a picture of what is happening.

5. Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

(a)
$$x_{n+1} = -16 + 6x_n + \frac{12}{x_n} \quad \alpha = 2$$

$$(b) \quad x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2} \quad \alpha = 3^{1/3}$$

$$(c) \quad x_{n+1} = \frac{12}{1+x_n} \quad \alpha = 3$$

6. Consider the mapping $g(x) = cx(1-x)$.

(a) Show that for $0 \leq c \leq 4$, g maps $[0, 1]$ into itself.

(b) Show that if $1 < c < 3$, g has a positive fixed point which is attracting.

(c) For $c = 3.2$ investigate the dynamics of the iteration $x_{n+1} = g(x_n)$. Certain numbers play a significant role here. Identify them as fixed points of some mapping.

7 A long conducting rod of diameter D meters and electrical resistance R per unit length is in a large enclosure whose walls (far away from the rod) are kept at temperature $T_s^\circ C$. Air flows past the rod at temperature T_∞ . If an electrical current I passes through the rod, the temperature of the rod eventually stabilizes to T , where T satisfies

$$Q(T) = \pi Dh(T - T_\infty) + \pi D\epsilon\sigma(T^4 - T_s^4) - I^2R = 0$$

where

$$\sigma = \text{Stefan-Boltzman constant} = 5.67 \cdot 10^{-8} \text{Watts/meter}^2 \text{Kelvin}^4,$$

$$\epsilon = \text{rod surface emissivity} = 0.8,$$

$$h = \text{heat transfer coefficient of air flow} = 20 \text{Watts/meter}^2 \text{Kelvin},$$

$$T_\infty = T_s = 25^\circ C,$$

$$D = 0.1 \text{ meter},$$

$$I^2R = 100.$$

Use FZERO to find the steady state temperature T of the rod.