ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #4 Due March 12, 2001

Readings: *Chapra & Canale* , Sections 6.5, 11.1, 11.2.

1. Consider the linear system $A\mathbf{x} = \mathbf{b}$, and let $\mathbf{x}^*$ be the computed solution (the number produced by the computer). The *error* in $\mathbf{x}^*$ is defined to be

$$\mathbf{e} = \mathbf{x} - \mathbf{x}^*,$$

and the *residual* is defined to be

$$\mathbf{r} = \mathbf{b} - A\mathbf{x}^*.$$

There are two principles concerning the accuracy of the solution $\mathbf{x}^*$ produced by Gaussian elimination with partial pivoting (The method used in $A\backslash\mathbf{b}$)

1. The residual is nearly always small. For a moderate sized problem the size of the residual is of the order of machine epsilon, $eps \approx 2.22 \times 10^{-16}$.

But this does not guarantee an accurate solution. The is a quantity called the *condition number* of A (given in MATLAB by cond(A)) which measures how close A is to being singular. If cond(A) is large we say that A is *ill-conditioned*. The second principle is:

2. If $\text{cond}(A) \approx 10^t$, the error is about $10^t \times eps \approx 10^{t-16}$ i.e. we have lost approximately t digits of accuracy.

The $n \times n$ matrix H_n whose elements are given by $h_{i,j} = 1/(i+j-1)$ is very ill-conditioned if n is large; it is called the *Hilbert Matrix*. If can be generated in MATLAB with the command **hilb(n)**. Let A be the Hilbert matrix of size 10. Let **x=ones(10,1)**, and let $\mathbf{b} = A\mathbf{x}$. Now use the backslash operator to solve the system $A\mathbf{x} = \mathbf{b}$, obtaining $\mathbf{x}^*$. Since we know $\mathbf{x}$ exactly, we can compute $\mathbf{e}$. Do this and also compute $\mathbf{r}$. What are the sizes of these vectors ? Are the above principles satisfied for this example ? Repeat with $n = 15$.

Use figure 11.2 on p.289 of *Chapra & Canale* to constuct a tridiagonal solver in MATLAB for the next three problems.

2. Problem 11.1, p.307 *Chapra & Canale* .

3. Problem 11.2, p.307 *Chapra & Canale* .

4. Problem 11.3, p.307 *Chapra & Canale* .

In the next two problems work interactively in MATLAB . Iterate until two susseccive iterations agree in the long format.

5. Problem 11.7, p.308 *Chapra & Canale* .

6. Problem 11.8, p.308 *Chapra & Canale* .

7. Solve the system

$$x^2 + xy^3 = 9 \qquad 3x^2y - y^3 = 4$$

using Newton's method for nonlinear systems. Use each of the initial guesses $(x_0, y_0) = (1.2, 2.5), (-2, 2.5), (-1.2, -2.5), (2, -2.5)$. Observe which root to which the method converges,the number of iterates required, and the speed of convergence. Write a MATLAB function with the initial guess as input. Be sure to include an appropriate stopping criterion and limit the number of iterations.