ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #5 Due March 28, 2001

Readings: *Chapra & Canale* , Sections 18.1, 18.2, 18.3.

1. (MATLAB) Let $f(x) = \sin x$ We wish to find $P_6(x)$, the polynomial of degree ≤ 6 interpolating $f(x)$ at $x_i = i\pi/3, \ i = 0, 1, \ldots, 6$. Find $P_6(x)$ in the form
$$P_6(x) = a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$$
by solving the system
$$a_6x_i^6 + a_5x_i^5 + a_4x_i^4 + a_3x_i^3 + a_2x_i^2 + a_1x_i + a_0 = f(x_i), \ i = 0, 1, \ldots 6.$$
(Use the MATLAB backslash operator.) Develop a function m-file for evaluating $P_6(x)$ using Horner's method. Plot $f(x)$ and $P_6(x)$ on the same graph and also $f(x) - P_6(x)$ for $0 \leq x \leq 2\pi$.

2. Problem 1, p.505, *Chapra & Canale* .

3. Problem 2, p.505, *Chapra & Canale* .

4. Problem 3, p.505, *Chapra & Canale* .

5. Consider the Lagrange polynomials $L_i(x), i = 0, 1, 2$ that are used for quadratic interpolation at the points x_0, x_1 and x_2. Define
$$g(x) = L_0(x) + L_1(x) + L_2(x) - 1.$$
(a) Show that g is a polynomial of degree ≤ 2.
(b) Show that $g(x_k) = 0$ for $k = 0,\ 1,\ 2$.
(c) Show that $g(x) = 0$ for all x. *Hint.* Use the fundamental theorem of algebra.

6. Let $L_0(x), L_1(x), \ldots, L_n(x)$ be the Lagrange polynomials based on the $n + 1$ points $x_0, x_1, \ldots, x_n$. Show that for every real x
$$\sum_{i=0}^{n} L_i(x) = 1.$$

7. Consider the function $f(x) = \sin x$ on the interval $[0, 1]$. Use the error bound stated in class to determine a step size h so that the error in linear interpolation is $< 5 \times 10^{-7}$.

8. The Runge function is
$$r(x) = \frac{1}{1 + x^2}, \quad -5 \leq x \leq 5.$$
(a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n + 1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?
(b) Repeat part (a) but now use the interpolation points
$$x_j = 5 \cos \frac{(2j - 1)\pi}{2n + 2}, \quad j = 1, \ldots, n + 1.$$
What difference do you observe ?