ENEE 241/MATH 242 Dr. Wolfe ASSIGNMENT #6 Due April 4, 2001

Readings: *Chapra & Canale,* Sections 18.6.3, 17.1, 17.2.

1. Consider the function S(x) defined as

$$S(x) = \begin{cases} 28 + 25x + 9x^2 + x^3, & -3 \le x \le -1, \\ 26 + 19x + 3x^2 - x^3, & -1 \le x \le 0, \\ 26 + 19x + 3x^2 - 2x^3, & 0 \le x \le 3, \\ -163 + 208x - 60x^2 + 5x^3, & 3 \le x \le 4. \end{cases}$$

   Show that $S(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. (A natural cubic spline is a spline $S(x)$ which satisfies $S''(x_1) = S''(x_N) = 0$) Be sure to state explicitly each of the properties of $S(x)$ which are necessary for this to be true.

2. Compute the natural cubic spline interpolating the data points $(-1, 1), (0, 0)$ and $(1, 2)$. Do it directly without reference to the constuction given in class. This involves the determination of 8 coefficients but it isn't that hard. Matching at the middle knot quickly reduces the number of unknown coefficients from 8 to 4. Applying the condition that it's a natural cubic spline enables you to eliminate 2 more coefficients, while fitting to the end points will enable you to find all the coefficients by solving a 2×2 system.

3. (MATLAB) Observed values for the thrust (T) versus time (t) curve of a model rocket are

t	0.00	0.05	0.10	0.15	0.20
T	0.0	1.0	5.0	15.0	33.5
t	0.30	0.40	0.50	0.60	0.70
T	33.0	16.5	16.0	16.0	16.0
t	0.80	0.85	0.90	0.95	1.00
T	16.0	16.0	6.0	2.0	0.0

   (a) Use the MATLAB functions POLYFIT and POLYVAL to find and plot the 14^{th} degree polynomial interpolating this data.

   (b) Use the MATLAB function SPLINE to find and plot the cubic spline interpolating the data.

   (c) Which function do you think does a better job of interpolating the data ? Why ? Suppose we also observe $T(0.25) = 38.0,\ T(0.65) = 16.0$. What values do the interpolation functions give in each case ? Compare the results with the observed values.

4. Find the best least squares fit by a linear function $y = Ax + B$ to the data points $(-2, 1)$, $(-1, 2)$, $(0, 3)$, $(1, 3)$, $(2, 4)$. Plot your linear function along with the data points.

5. Find the best least squares fit by a quadratic function $y = Ax^2 + Bx + C$ to the data points $(-2, 10)$, $(-1, 1)$, $(0, 0)$, $(1, 2)$, $(2, 9)$. Plot the points and the parabola.

6. The vapor pressure P of water (in bars) as a function of temperature $T\,(^{\circ}C)$ is

T	0	10	20	30
P(T)	.006107	.012277	.023378	.042433
T	40	50	60	70
P(T)	.073774	.12338	.19924	.31166
T	80	90	100	110
P(T)	.47364	.70112	1.01325	1.22341

We wish to fit this data to a quadratic polynomial $P = \beta_0 + \beta_1 T + \beta_2 T^2$ in the sense of least squares. The MATLAB commands POLYFIT and POLYVAL give you exactly what you need. Plot P along with the points.

7. (MATLAB) Let $\mathbf{x} = \mathbf{0} : .\mathbf{1} : \mathbf{10}$; $\mathbf{y} = \mathbf{2x} + \mathbf{4}$. Of course the graph of y is a straight line. Let's introduce some noise into the system. So write $\mathbf{y} = \mathbf{y} + .\mathbf{2} * \mathbf{rand}(\mathbf{1}, \mathbf{101}) - .\mathbf{1}$. Now use POLYFIT to fit the noisy data to a straight line. Do this three or four times. What do you observe ?