ENEE 241/MATH 242   Dr. Wolfe   ASSIGNMENT #9   Due May 2, 2001

Readings: *Chapra & Canale* Sections 22.2, 22.3.

The first 6 problems concern

$$I = \int_0^1 e^{-x^2}\,dx.$$

The Fundamental Theorem of Calculus is of no help in evaluating I but it is known that

$$I = .74682413281243.$$

1. Use your programs TRAP and SIMP from the previous assignment to calculate approximations to I. Let $n = 2, 4, 8, 16, \ldots, 512$. Compute the errors.

2. Use the corrected trapezoidal rule to the integral (for the same values of n as in problem 1). How do the results compare with the corresponding results for Simpson's rule ?

3. Use the MATLAB function QUAD or QUADL on I. Use several different values of TOL.

4. Compute the Romberg approximation $I_{16}^{(4)}$ to I. Here $I_j^{(0)} = T_j$, the j-panel trapezoidal rule and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_{n/2}^{k-1}}{4^k - 1}$$

   for n a multiple of 2^k, $k \geq 1$. (You could write a script for this or just use MATLAB as a calculator.)

5. Compute the Gauss-Legendre approximations of I with $n = 2, 4, 6, 8$. I have supplied an m-file with the weights and nodes. Remember, you have to make a change of variable to transform the integal to one whose limits of integration are -1 and 1.

6. The 11 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x)\,dx \approx \sum_{i=0}^{10} w_i f(\frac{i}{10})$$

   with the w_i determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots x^{10}$.
   (a) Use MATLAB to find the weights w_i.
   (b) Apply the rule to I. Note the error.

Problems 7 & 8 refer to the integral

$$J = \int_2^3 \frac{dx}{5 - x}$$

7. Determine N so that the N-panel trapezoidal rule can be used to compute J with an accuracy of 5×10^{-9}.

8. Determine N so that the N-panel Simpson's rule can be used to compute J with an accuracy of 5×10^{-9}.