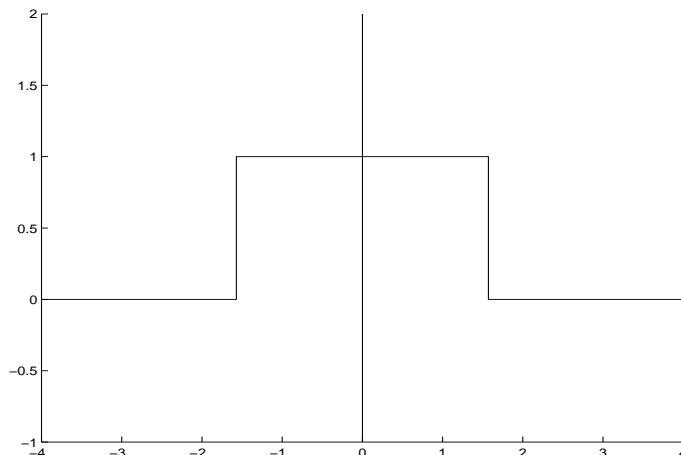


1, Let

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x \leq \frac{-\pi}{2} \\ 1 & \text{for } \frac{-\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

The graph of  $f$  is shown below. Find the Fourier series representation of  $f$ .



2. Suppose that  $f(x)$  is a smooth function such that

$$\int_2^5 f(x) dx = 6$$

Suppose the result of applying the trapezoid rule with 20 panels to approximate this integral is  $T_{20} = 5.84$ . Give an estimate of  $T_{40}$ , the result of applying the trapezoid rule with 40 panels. Explain.

3. (10 points) Suppose we use Simpson's rule with 400 panels to approximate

$$I = \int_0^1 (x^3 + 3x^2 - x) dx.$$

Assuming no roundoff error, what will the result be? Explain.

4. Let

$$I = \int_0^1 \frac{1}{1+x} dx = \ln 2 = .69314718$$

Compute approximations to  $I$  using

- The 4 panel trapezoid rule.
- The 4 panel Simpson's rule.
- The 4 panel corrected trapezoid rule.

Which method gives the best result?