

AMSC/CMSC 460 SUMMER 2003

SAMPLE MIDTERM EXAM

1. Let N be a positive integer. Consider the following MATLAB script:

```
y = 0;
for i = 1 : N
y = y + (1/N);
end
y
```

- (a) What would the result of the computation be in exact arithmetic ?
(b) When the script was actually run with $N = 100,000 (= 10^5)$ the result was $y = .9999999999808$. When it was run with $N = 131,072 (= 2^{17})$ the result was $y = 1$. Explain these results.
2. Let

$$A = \begin{pmatrix} 4 & 6 \\ 6 & 13 \end{pmatrix}.$$

A is symmetric, positive definite.

- (a) Find a lower triangular matrix L with positive diagonal entries such that $A = LL^T$ (Cholesky factorization).
(b) Let $\mathbf{b} = (-2, 1)^T$. Use the factorization of part (a) to solve $A\mathbf{x} = \mathbf{b}$ by forward elimination and back substitution.
3. Let $f(x) = x^3$.
- (a) Find the quadratic polynomial $p_2(x)$ interpolating $f(x)$ at $x_0 = 0, x_1 = 1, x_2 = 2$.
(b) Find $P(x)$, the piecewise linear interpolant to $f(x)$ with breakpoints x_0, x_1, x_2 .
(c) Find the linear function $L(x)$ which best fits the data $(0, 0), (1, 1), (2, 8)$ in the sense of least squares. Plot $f(x)$ and $L(x)$ on the same graph.
(d) Which of the functions found in parts (a),(b), and (c) do you think does the best job of approximating $f(x)$ on $[0, 2]$? Explain.

4. Let

$$S(x) = \begin{cases} x + 1 & -2 \leq x \leq -1, \\ x^3 - 2x - 1 & -1 \leq x \leq 1, \\ x - 3 & 1 \leq x \leq 2 \end{cases}$$

Is $S(x)$ a natural cubic spline ? Explain.

- 5.

- (a) Find a polynomial $p(x)$ of degree ≤ 2 satisfying $p(0) = 0, p(1) = 1, p'(1/4) = 2$.
(b) There is a number $c, 0 < c < 1$, such that there is no polynomial $p(x)$ of degree ≤ 2 satisfying $p(0) = 0, p(1) = 1, p'(c) = 2$. Find c .

6. Let

$$I = \int_0^1 \frac{2}{1+x} dx = \ln 4 = 1.38629436$$

Compute approximations to I using

- (a) The 4 panel trapezoid rule.
- (b) The 4 panel Simpson's rule.
- (c) The 4 panel corrected trapezoid rule.

Which method gives the best result ?

7. Mark each of the following statements as true (T) or false (F).

- (a) A problem is ill-conditioned if its solution is highly sensitive to small changes in the problem data.
- (b) If x is any vector in \mathbf{R}^n then $\|x\|_1 \geq \|x\|_\infty$.
- (c) Any nonsingular matrix A can be factored as $A = LU$ with L lower triangular and U upper triangular.
- (d) When interpolating a continuous function by polynomials at equally spaced points on a given interval, the polynomial interpolants always converge to the function as the number of interpolation points increases.
- (e) Given $n + 1$ data points $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ with $x_0 < x_1 < \dots < x_n$ there is a unique cubic spline interpolating these points.