AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #3 Due March 17, 2004

1. Define $f(x)$ by
$$f(x) = \frac{1}{1+x^2}, \quad -5 \le x \le 5.$$
   (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $f(x)$ at $n+1$ equally spaced points, along with the graph of $f(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?
   (b) Repeat part (a) but now use the interpolation points
$$x_j = 5\cos\frac{(2j-1)\pi}{2n+2}, \quad j = 1, \ldots, n+1.$$
   What difference do you observe ?

2. Ex. 3.10, p.90, *Shampine, Allen & Preuss.*

3. For $f(x) = \tan(x)$ we are given that
$$f(0) = 0,\ f'(0) = 1,\ f(\pi/4) = 1,\ f'(\pi/4) = 2.$$
   Calculate an approximation to $f(\pi/8)$ using cubic Hermite interpolation. Compare the result with $f(\pi/8) = \sqrt{2} - 1$.

4. Ex. 3.16, p.98, *Shampine, Allen & Preuss.*

5.
   (a) Determine all values of a, b, c, d, e for which the following function is a cubic spline:
$$S(x) = \begin{cases} a(x-2)^2 + b(x-1)^3, & 0 \le x \le 1, \\ c(x-2)^2, & 1 \le x \le 3, \\ d(x-2)^2 + e(x-3)^3, & 3 \le x \le 4, \end{cases}$$
   (b) Determine the values of the parameters so that the cubic spline interpolates the data $(0, 26), (1, 7), (4, 25)$.

6. Ex. 3.22, p.115, *Shampine, Allen & Preuss.* Use the MATLAB function SPLINE. You should produce a graph.

7. Repeat the previous problem using $p_{10}(x)$, the polynomial of degree ≤ 10 which interpolates f at the given points. How does it do compared with the spline ?

8. Take the data of problem 6 and find and plot the polynomials of degree 2, 3 and 4 which best fit this data in the sense of least squares. The MATLAB functions POLYVAL and POLYFIT give you exactly what you need.

9. For the data of problem 6 find the cubic polynomial $p_3(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3$ interpolating the data in the sense of least squares by constructing the 11×4 data matrix A and finding the vector $(p_0, p_1, p_2, p_3)^T$ in four different ways:

(a) By using the backslash operator.
(b) By forming and solving the normal equations. Note the condition number of the matrix $A^T A$.
(c) By using the QR decomposition.
(d) By using the Singular-Value Decomposition.

All of this is quite easy in MATLAB. Compare with the values found in problem 8.