AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #4 Due April 12, 2004

1. Write a MATLAB program to evaluate $I = \int_a^b f(x)\,dx$ using the trapezoidal rule with n subdivisions, calling the result I_n. Use the program to calculate the following integrals with $n = 2, 4, 8, 16, \ldots, 512$.

$$\text{(a)} \quad \int_0^1 x^4 \arctan(x)\,dx \qquad \text{(b)} \quad \int_0^1 x^{2/3}\,dx$$

The exact value of the integral in (a) is $\frac{\pi+1-2\ln(2)}{20}$. Try to arrange your work so that you never compute the value of the integrand at any point more than once. Analyze emperically the rate of convergence of I_n to I by calculating the ratios

$$R_n = \frac{I_{2n} - I_n}{I_{4n} - I_{2n}} \text{ and } p_n = \frac{\log(R_n)}{\log(2)}$$

In part (b) compute the extrapolated approximation to I,

$$\tilde{I} = I_{4n} - \frac{(I_{4n} - I_{2n})^2}{(I_{4n} - I_{2n}) - (I_{2n} - I_n)}$$

for $n = 128$.

2. Repeat problem 1 using Simpson's rule.

3. Apply the corrected trapezoidal rule to the integral in problem 1(a). Compare the results with those of problem 2 for Simpson's rule.

4. Use Gauss-Legendre integration with $n = 2, 4, 8$ nodes to the integrals of problem 1. Compare the results with those for the trapezoidal and Simpson methods.

5. Find approximate values of the integrals in problem 1 by computing the Romberg integral $I_{32}^{(5)}$ where $I_n^{(0)}$ is the n-panel trapezoid approximation and

$$I_n^{(k)} = \frac{4^k I_n^{(k-1)} - I_{n/2}^{(k-1)}}{4^k - 1}$$

for n divisible by 2^k.

6. Use the MATLAB function QUADL to find approximate values of the integrals 1(a) and 1(b).

7. Ex. 5.1, p.183, *Shampine, Allen & Preuss.*

8. The 10 point Newton-Cotes integration rule on $[0, 1]$ is

$$\int_0^1 f(x)\,dx \approx \sum_{i=0}^{9} w_i f(\frac{i}{9})$$

with the w_i determined by requiring that the rule be exact for $f(x) = 1, x, x^2, \ldots x^9$.

(a) Use MATLAB to find the weights w_i.

(b) Apply the rule to the integrals in 1(a) and 1(b). Note the errors.

9. We wish to estimate the value of

$$I = \int_0^\infty x^{3/2} e^{-x}\, dx = \frac{3}{4}\sqrt{\pi}$$

(a) Truncate the integral and use QUADL on the finite part.

(b) Try the transformation $x = -\ln t$ on this integral and use QUADL on the new integral. (QUADL will complain but will do it.)

(c) Use the 2, 4 and 8 point Gauss-Laguerre rules to estimate the integral. compare your results with parts (a) and (b) above.

10. In a standard shell and heat exchanger hot vapor condenses on the tube, maintaining a constant temperature T_s. If the input is at temperature T_1 and the output must be at temperature T_2, then the length of tube required is given by

$$L = \frac{m}{\pi D} \int_{T_1}^{T_2} \frac{c_\rho dT}{h(T_s - T)}.$$

(All quantities must be in consistent units.) Here T is the temperature in °F.
$T_1 = 0$°F is the inlet temperature.
$T_2 = 180$°F is the desired outlet temperature.
$T_s = 250$°F is the condensate temperature.
m is the fluid flow rate $= 45,000$ lb/hr.
D is the diameter of the tube $= 1.032$ in.
c_ρ is the specific heat of the fluid $= (0.53 + 0.00065T)$ BTU/(lb°F).
h is the local heat transfer coefficient $= \frac{0.023k}{D}(\frac{4m}{\pi D\mu})^{0.8}(\frac{\mu c_\rho}{k})^{0.4}$.
k is the thermal conductivity of the fluid $= 0.153$ BTU/(hr ft°F).
μ is the viscosity of the fluid and has units lb/(ft hr). μ varies with temperature so that

T	0	50	100	150	200
μ	242	82.1	30.5	12.6	5.57

Use spline interpolation to define μ for other values of T and calculate the required length of the heat exchanger.

item You will need to use the MATLAB functions SPLINE and QUADL. The answer is about 158.7 ft.