AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #5 Due April 26, 2004

1.
   (a) Implement the bisection method in MATLAB to find the smallest positive root of
   $$e^{-x} = \tan x \tag{1}$$
   (b) Solve (1) using the secant method. (Use either a calculator or MATLAB.)

2. Write a MATLAB function Newton(f, df, x, tol) to implement Newton's method. You need to supply functions $f(x)$ and $df(x)(f'(x))$. The input x is the initial guess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.
   (a) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten <u>positive</u> solutions of
   $$x = \cot x.$$
   Note: The careful selection of x is critical.
   (c) Try the function on the double root $x = 2$ of
   $$x^3 - x^2 - 8x + 12 = 0.$$
   Use $x = 3$ and $tol = 10^{-6}$. What is the rate of convergence ?
   (d) Newton can be used to find complex roots also. By starting with a non-real initial guess, find the complex roots of
   $$x^3 + 2x - 5 = 0.$$

3. Write down two fixed point procedures for finding a zero of the function $f(x) = 2x^2 + 6e^{-x} - 4$. Check that they converge.

4. Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

   (*a*) $$x_{n+1} = -16 + 6x_n + \frac{12}{x_n} \qquad \alpha = 2$$

   (*b*) $$x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2} \qquad \alpha = 3^{1/3}$$

   (*c*) $$x_{n+1} = \frac{12}{1 + x_n} \qquad \alpha = 3$$

5. Ex. 4.13. p.156, *Shampine, Allen & Pruess.* Use the MATLAB code FZERO.

6. Ex. 4.16. p.156, *Shampine, Allen & Pruess.* Use the MATLAB code FZERO.

7. Solve the system

$$x^2 + xy^3 = 9 \qquad 3x^2y - y^3 = 4$$

   using Newton's method for nonlinear systems. Use each of the initial guesses $(x_0, y_0) = (1.2, 2.5), (-2, 2.5), (-1.2, -2.5), (2, -2.5)$. Observe which root to which the method converges,the number of iterates required, and the speed of convergence. Write a MATLAB function with the initial guess as input. Be sure to take advantage of the fact that MATLAB works with vectors.

8. Ex. 5.12. p.191, *Shampine, Allen & Pruess.* Use QUADL. You will need the MATLAB command GLOBAL to pass the value c into the integrand.

9. Consider the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 \\ -1 & 4 & -1 & 0 & -1 \\ 0 & -1 & 4 & -1 & 0 \\ -1 & 0 & -1 & 4 & -1 \\ 0 & -1 & 0 & -1 & 4 \end{pmatrix}$$

   and $\mathbf{b} = (-2, -1, 6, 7, 14)'$. Solve the system using

(a) The Cholesky factorization of A (MATLAB: CHOL)
(b) Jacobi iteration.
(c) Gauss-Seidel iteration. (The MATLAB command TRIL might be useful.)