AMSC/CMSC 460 SPRING 2004

SAMPLE HOUR EXAM

1. Suppose you have a computer which carries only 4 decimal digits and rounds. It is desired to compute $f(x) = \frac{e^x - x - 1}{x^2}$ at $x = .001$. The value of $e^{.001}$ correctly rounded to 4 decimal places is 1.001.
   (a) Using the definition of f and the above value of $e^{.001}$ what result would the computer give for $f(.001)$?
   (b) By using Taylor's theorem and 4 digit arithmetic, find the correct value of $f(.001)$ rounded to 4 digits.

2. Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 4 \\ 3 & 6 & 7 \end{pmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}.$$

   (a) Factor A as $A = LU$ with L lower triangular and U upper triangular.
   (b) What happens if we try to use the decomposition found in (a) to solve $A\mathbf{x} = \mathbf{b}$? What do you conclude about this equation ?

3. Let $\|\mathbf{x}\|$ be a norm defined on $\mathbf{R}^n$.
   (a) Define the matrix norm $\|A\|$, defined on the set of $n \times n$ matrices, associated with this norm.
   (b) Define the *condition number*, cond(A), associated with the above norm.
   (c) Suppose cond$A = 1000$ and $A\mathbf{x} = \mathbf{b}$ is solved on a computer to give a result in which all components of the residual are less than 10ϵ. If $1 \leq b_i \leq 10$, $i = 1, \ldots, n$ and the computed $x_i \approx i$, $i = 1, \ldots, n$, what bounds can you put on the actual errors in the x_i. Assume the ∞-norm is used.

4.
   (a) Find the quadratic polynomial $p_2(x)$ which interpolates the function $f(x) = \cos\frac{(x-1)\pi}{3}$ at $x = 0$, $x = 1$, $x = 2$. Give the Lagrange form, a Newton form, and the standard form of $p_2(x)$.
   (b) Compute $p_2(1/2)$ by Horner's method. Compare the actual value of $f(1/2) - p_2(1/2)$ with the theoretical error bound for quadratic interpolation.

5. Let

$$s(x) = \begin{cases} x^3 - 3x^2 + 2x + 1, & 1 \leq x \leq 2 \\ -x^3 + 9x^2 - 22x + 17, & 2 \leq x \leq 3 \end{cases}$$

   Is $s(x)$ a cubic spline ? Is it a natural cubic spline ?