AMSC/CMSC 460 Sample Second Hour Exam

1. (15 points) Find the best least squares fit by a linear function $y = \beta_0 + \beta_1 x$ to the data points $(-1, 0), (0, 2), (1, 5), (2, 8)$. Plot your linear function along with the data points in the xy plane.

2. (35 points) Let
$$I = \int_{-1}^{1} \frac{1}{x+4}\, dx = .5108256238$$
Compute approximations to I using
   (a) The 4 panel trapezoid rule.
   (b) The 4 panel Simpson's rule.
   (c) The two point Gauss-Legendre rule. (Recall that the nodes for this are $\pm\frac{1}{\sqrt{3}}$.)
   Which method gives the best result ?

3. (35 points) Let $f(x) = x - \frac{2}{x}$. The positive root of f is $\sqrt{2}$.
   (a) Let $x_0 = 2, x_1 = 1$. Use the secant method to find two new approximations to $\sqrt{2}$, x_2 and x_3.
   (b) What is the formula for Newton's method in this case ?
   (c) Will the iteration scheme
   $$x_{n+1} = 2.5x_n - .75x_n^3, \ x_0 \text{ given}, 1 \le x_0 \le 2$$
   converge ? Explain.
   (d) Find c so that the iteration scheme $x_{n+1} = x_n + cf(x_n)$ is convergent (given an appropriate starting value x_0).

4. (15 points) We wish to solve the system
$$x^2y + y^2 - x = 2, \quad 3x + y - x^2y^2 = 2$$
by Newton's method. If $(x_0, y_0) = (1, 1)$ what is (x_1, y_1) ? Do you think (x_1, y_1) is closer to a root than (x_0, y_0) ?