CMSC/MAPL 460 Dr. Wolfe ASSIGNMENT #5 Due April 25, 2001

1.
   (a) Implement the bisection method in MATLAB to find the smallest positive root of
   $$e^{-x} = \sin x \tag{1}$$
   (b) Solve (1) using the secant method. (Use either a calculator or MATLAB.)

2. Write a MATLAB function Newton(x, tol) to implement Newton's method. You need to write function files $f(x)$ and $df(x)(f'(x))$. The input x is the initial guess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.
   (a) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten positive solutions of
   $$x = \tan x.$$
   (Zero is not a positive number.) Note: The careful selection of x is critical.
   (c) Try the function on the double root $x = 1$ of
   $$x^3 + 3x^2 - 9x + 5 = 0.$$
   Use $x = 2$ and $tol = 10^{-6}$. What is the rate of convergence ?

3. Let
   $$g(x) = \frac{1}{2}\sqrt{10 - x^3}.$$
   (a) Show that the equation $g(x) = x$ has exactly one solution, α.
   (b) Find an interval $[a, b]$ such that for every $x_0 \in [a, b]$ the iteration $x_{n+1} = g(x_n)$ converges to α.
   (c) Find α using fixed point iterations.
   (d) Find α using a combination of fixed point iterations and Aitken extrapolation.

4. Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

   (a) $$x_{n+1} = -16 + 6x_n + \frac{12}{x_n} \qquad \alpha = 2$$

   (b) $$x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2} \qquad \alpha = 3^{1/3}$$

(c) $$x_{n+1} = \frac{12}{1 + x_n} \qquad \alpha = 3$$

5. Given below is a table of iterates from a linearly convergent iteration $x_{n+1} = g(x_n)$. Estimate (a) the rate of linear convergence, (b) the fixed point α, and (c) the error $\alpha - x_5$.

n	x_n
0	1.0949242
1	1.2092751
2	1.2807917
3	1.3254943
4	1.3534339
5	1.3708962

6. Ex. 4.11. p.155, *Shampine, Allen & Pruess*. Use the MATLAB code Zero.

7. Ex. 4.17. p.156, *Shampine, Allen & Pruess*. Use the MATLAB code Zero.

8. Solve the system
$$x^2 + xy^3 = 9 \qquad 3x^2y - y^3 = 4$$
using Newton's method for nonlinear systems. Use each of the initial guesses $(x_0, y_0) = (1.2, 2.5), (-2, 2.5), (-1.2, -2.5), (2, -2.5)$. Observe which root to which the method converges,the number of iterates required, and the speed of convergence. Write a MATLAB function with the initial guess as input. Be sure to take advantage of the fact that MATLAB works with vectors.