

1. Run the following MATLAB programs and explain the output.

<code>x = 1;</code>	<code>x = 1;</code>	<code>x = 1;</code>
<code>while x + 1 > 1</code>	<code>while x > 0</code>	<code>while x < Inf</code>
<code>y = x;</code>	<code>y = x;</code>	<code>y = x;</code>
<code>x = x/2;</code>	<code>x = x/2;</code>	<code>x = 2 * x;</code>
<code>end</code>	<code>end</code>	<code>end</code>
<code>y</code>	<code>y</code>	<code>y</code>

Assume $\beta = 2$. What can you infer about the machine constants t, U and L ? (The last two constants are called M and m in *Shampine et al.*)

2. Using four digit arithmetic, add the following numbers, first in ascending order (from smallest to largest) and then in descending order. In doing so round off the partial sums to four significant figures. Compare your results with the correct sum $x = 0.107101023 \text{ E}5$. (En means 10^n .)

0.1580 E0	0.4288 E1	0.7767 E3
0.2653 E0	0.6266 E2	0.7889 E3
0.2581 E1	0.7555 E2	0.8999 E4

3. Let

$$f(x) = \frac{\ln(1-x) + xe^{x/2}}{x^3}.$$

Use MATLAB to compute $f(x)$ for $x = 10^{-m}$, $m = 1, 2, \dots, 12$. According to the theory what is $\lim_{x \rightarrow 0} f(x)$? For x near zero what is a better way to compute $f(x)$? (Hint: Use Taylor's theorem on the numerator.)

4. How can values of the function $f(x) = \sqrt{x+2} - \sqrt{x}$ be computed accurately when x is large?
5. Suppose a computer carries three decimal digits and rounds. If x and y are machine numbers, define the machine version of addition $x \oplus y$ to be the result of adding x and y and rounding to three digits. For example

$$49.3 \oplus 57.4 = 107.$$

Define machine multiplication $x \otimes y$ similarly. For example,

$$1.23 \otimes 4.86 = 5.98.$$

Construct examples to show that, in general, the following statements are not true:

- (a) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$,
- (b) $(x \otimes y) \otimes z = x \otimes (y \otimes z)$,
- (c) $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$.

6. The **error function** is an important function in many branches of applied mathematics. It is defined by an integral.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt.$$

The integral cannot be expressed in terms of more elementary functions. It is a library function in MATLAB. The Taylor series for the error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}.$$

The series converges for all x . Write a MATLAB function m-file to evaluate $\operatorname{erf}(x)$ using this series. Use as many terms in the series as are necessary so that the first neglected term does not alter the accumulated sum when it is added to it in floating point arithmetic. Since this is an alternating series, the error caused by truncating the infinite sum will then be less than the roundoff error. Investigate the effect of roundoff error by comparing the computed sum with the value given by the MATLAB function `erf`. Try $x = 0.5, 1.0, 2.0, 5.0, 7.0, 10.0$. Explain your results. *Hint:* The main loop of your program might look like this:

```
while s ~= so
    so = s;
    n = n + 1;
    t = -xsq * t * (2.0 * n - 1.0) / (n * (2 * n + 1));
    s = s + t;
end
```

What values should be assigned to t, n, s and xsq before entering the loop? Do not forget the factor $2/\sqrt{\pi}$.

7. Ex. 1.2, p.11, *Shampine, Allen & Pruess*
8. Ex. 1.9, p.23, *Shampine, Allen & Pruess*
9. Ex. 1.16, p.29, *Shampine, Allen & Pruess*