CMSC/MAPL 460 Dr. Wolfe ASSIGNMENT #3 Due March 26, 2001

1. The Runge function is

$$r(x) = \frac{1}{1+x^2}, \quad -5 \le x \le 5.$$

   (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n+1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?

   (b) Repeat part (a) but now use the interpolation points

$$x_j = 5\cos\frac{(2j-1)\pi}{2n+2}, \quad j = 1, \ldots, n+1.$$

   What difference do you observe ?

2. Ex. 3.4, p.90, *Shampine, Allen & Preuss.* (Just use the MATLAB backslash operator.)

3. Ex. 3.10, p.90, *Shampine, Allen & Preuss.*

4. For $f(x) = \sinh x$ we are given that

$$f(0) = 0, \; f'(0) = 1, \; f(1) = 1.1752, \; f'(1) = 1.5431.$$

   Calculate an approximation to $f(0.5)$ using cubic Hermite interpolation. Compare the result with $f(0.5) = .5211$.

5. Ex. 3.15, p.98, *Shampine, Allen & Preuss.*

6. Consider the function S(x) defined as

$$S(x) = \begin{cases} 28 + 25x + 9x^2 + x^3, & -3 \le x \le -1, \\ 26 + 19x + 3x^2 - x^3, & -1 \le x \le 0, \\ 26 + 19x + 3x^2 - 2x^3, & 0 \le x \le 3, \\ -163 + 208x - 60x^2 + 5x^3, & 3 \le x \le 4. \end{cases}$$

   Show that $S(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. (A natural cubic spline is a spline $S(x)$ which satisfies $S''(x_1) = S''(x_N) = 0$) Be sure to state explicitly each of the properties of $S(x)$ which are necessary for this to be true.

7. Ex. 3.22, p.115, *Shampine, Allen & Preuss.* Use the MATLAB function SPLINE.

8. Take the data set from Ex. 3.22 and find and plot the polynomials of degree 2, 3 and 4 which best fit this data in the sense of least squares. The MATLAB functions POLYVAL and POLYFIT give you exactly what you need.

9. For the data of problem 7 find the cubic polynomial $p_3(x) = p_0 + p_1x + p_2x^2 + p_3x^3$ interpolating the data in the sense of least squares by constructing the 11×4 data matrix A and finding the vector $(p_0, p_1, p_2, p_3)^T$ in four different ways:

   (a) By using the backslash operator.

   (b) By forming and solving the normal equations. Note the condition number of the matrix A^TA.

   (c) By using the QR decomposition.

   (d) By using the Singular-Value Decomposition.

   All of this is quite easy in MATLAB. Compare with the values found in problem 8.