# CMSC/MAPL 460 Sample Second Hour Exam

1. We wish to fit the data (0,1), (1,3), (2,7), (3,10), (4,20) to a function of the form

$$f(x) = a + bx + ce^x$$

in the sense of least squares. Find an equation for the coefficients $a, b$ and $c$. Do not do any computations.

2. Suppose we use Simpson's rule with 400 panels to approximate

$$I = \int_0^1 (x^3 + 3x^2 - x)dx.$$

Assuming no roundoff error, what will the result be ? Explain.

3. Compute an approximation $T_4$ to $I = \int_{-1}^{1} |x|\, dx$ using the trapezoid rule with 4 panels. Compute the error $I - T_4$. Explain your results.

4.
   (a) Find constants $\alpha_1$ and $\alpha_2$ such that the integration rule

$$I = \int_0^h f(x)\, dx \approx \alpha_1 f(h/4) + \alpha_2 f(3h/4) = Q$$

   is exact for all linear polynomials.

   (b) Is the above rule exact for quadratics ?

   (c) Given that the error of the rule is of the form

$$e = I - Q = cf^{(d)}(\zeta)h^{d+1}$$

   for smooth $f$, where $0 < \zeta < h$, find the integer $d$ and the constant $c$.

5. Let $\mathbf{f} : \mathbf{R^n} \to \mathbf{R^n}$ be a smooth function. What is Newton's method for solving $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ ? Be sure to define any terms you use.

6. The equation $3 \ln x = x$ has two real solutions, one near $x = 2$, the other near $x = 4.5$.
   (a) Rewrite the equation in the form $x = g(x)$ so that the fixed point iterations will converge to the root near $x = 2$. Find this root.
   (b) Rewrite the equation in the form $x = g(x)$ so that the fixed point iterations will converge to the root near $x = 4.5$. Find this root.

   Note: You may use an iteration based on Newton's method to find one but not both of these roots.