

CMSC/MAPL 460 SPRING 2001

SAMPLE HOUR EXAM

1. Suppose you have a computer which carries only 4 decimal digits and rounds. It is desired to compute  $f(x) = \frac{e^x - x - 1}{x^2}$  at  $x = .001$ . The value of  $e^{.001}$  correctly rounded to 4 decimal places is 1.001.
  - (a) Using the definition of  $f$  and the above value of  $e^{.001}$  what result would the computer give for  $f(.001)$  ?
  - (b) By using Taylor's theorem and 4 digit arithmetic, find the correct value of  $f(.001)$  rounded to 4 digits.
2. Consider the linear system  $A\mathbf{x} = \mathbf{b}$  where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 4 \\ 3 & 6 & 7 \end{pmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}.$$

- (a) Factor  $A$  as  $A = LU$  with  $L$  lower triangular and  $U$  upper triangular.
  - (b) What happens if we try to use the decomposition found in (a) to solve  $A\mathbf{x} = \mathbf{b}$  ? What do you conclude about this equation ?
3. Let  $\|\mathbf{x}\|$  be a norm defined on  $\mathbf{R}^n$ .
    - (a) Define the matrix norm  $\|A\|$ , defined on the set of  $n \times n$  matrices, associated with this norm.
    - (b) Define the *condition number*,  $\text{cond}(A)$ , associated with the above norm.
    - (c) Suppose  $\text{cond}A = 1000$  and  $A\mathbf{x} = \mathbf{b}$  is solved on a computer to give a result in which all components of the residual are less than  $10\epsilon$ . If  $1 \leq b_i \leq 10$ ,  $i = 1, \dots, n$  and the computed  $x_i \approx i$ ,  $i = 1, \dots, n$ , what bounds can you put on the actual errors in the  $x_i$ . Assume the  $\infty$ -norm is used.
  4.
    - (a) Find the quadratic polynomial  $p_2(x)$  which interpolates the function  $f(x) = \cos \frac{(x-1)\pi}{3}$  at  $x = 0$ ,  $x = 1$ ,  $x = 2$ . Give the Lagrange form, a Newton form, and the standard form of  $p_2(x)$ .
    - (b) Compute  $p_2(1/2)$  by Horner's method. Compare the actual value of  $f(1/2) - p_2(1/2)$  with the theoretical error bound for quadratic interpolation.

5. Let

$$s(x) = \begin{cases} x^3 - 3x^2 + 2x + 1, & 1 \leq x \leq 2 \\ -x^3 + 9x^2 - 22x + 17, & 2 \leq x \leq 3 \end{cases}$$

Is  $s(x)$  a cubic spline ? Is it a natural cubic spline ?