

1. Using four digit arithmetic, add the following numbers, first in ascending order (from smallest to largest) and then in descending order. In doing so round off the partial sums to four significant figures. Compare your results with the correct sum  $x = 0.114623141e + 5$ . (e+n means  $10^n$ .)

0.2673e+0	0.2875e+1	0.8153e+3
0.3548e+0	0.4478e+2	0.8786e+3
0.1997e+1	0.7214e+2	0.9646e+4

2. Let

$$f(x) = \frac{\ln(1+x) - e^x + 2 \cosh x - 1}{x^3}.$$

Use MATLAB to compute  $f(x)$  for  $x = 10^{-m}$ ,  $m = 1, 2, \dots, 8$ . According to the theory what is  $\lim_{x \rightarrow 0} f(x)$ ? For  $x$  near zero what is a better way to compute  $f(x)$ ? (Hint: Use Taylor's theorem on the numerator.)

3. Ex. 1.2, p.11, *Shampine, Allen & Preuss*.  
 4. Ex. 1.9, p.23, *Shampine, Allen & Preuss*.  
 5. The **error function** is an important function in many branches of applied mathematics. It is defined by an integral.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt.$$

The integral cannot be expressed in terms of more elementary functions. It is a library function in MATLAB. The Taylor series for the error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}.$$

The series converges for all  $x$ . Write a MATLAB function m-file to evaluate  $\operatorname{erf}(x)$  using this series. Use as many terms in the series as are necessary so that the first neglected term does not alter the accumulated sum when it is added to it in floating point arithmetic. Since this is an alternating series, the error caused by truncating the infinite sum will then be less than the roundoff error. Investigate the effect of roundoff error by comparing the computed sum with the value given by the MATLAB function `erf`. Try  $x = 0.5, 1.0, 2.0, 5.0, 7.0, 10.0$ . Explain your results. *Hint*: The main loop of your program might look like this:

```

while s ~= so
so = s;
n = n + 1;
t = -xsq * t * (2.0 * n - 1.0) / (n * (2 * n + 1));
s = s + t;
end

```

What values should be assigned to  $t, n, s$  and  $xsq$  before entering the loop ? Do not forget the factor  $2/\sqrt{\pi}$ . Before running the script, give the MATLAB command “format long”.

6. Ex. 1.13, p.24, *Shampine, Allen & Preuss*. . Show that the first recurrence gives values approaching  $\pi$  but ultimately fails, while you can use the second recurrence to compute  $\pi$  to full precision. (Again, you should be in the “format long” mode.)
7. Ex. 1.15, p.29 *Shampine, Allen & Preuss*.
8. Ex. 1.17, p.29, *Shampine, Allen & Preuss*.
9. Write a MATLAB script which takes as input  $n$  and plots both  $\sin(x)$  and

$$S_n(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

over the interval  $[0, 2\pi]$ . Try your script for several values of  $n$  and describe the results.