AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #6 Due July 26, 2002

1. Compute numerical approximations to the solution of the initial value problem

$$\frac{dy}{dx} = y + \cos x, \quad y(0) = 1$$

using the Euler, Improved Euler and Runge-Kutta methods. For Euler and Improved Euler take $h = 0.1\,, 0.05,\ 0.025$. For Runge-Kutta take $h = 0.2,\ 0.1$. Print out your results at $x = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0$. Compute the actual errors by comparing your results with the exact solution $y = 1.5e^x + .5\sin x - .5\cos x$. Discuss your results.

2. Ex. 6.27 p.249, *Shampine, Allen & Preuss.*

3. The following differential equations describe the motion of a body in orbit about two much heavier bodies. An example would be a spacecraft in an earth-moon orbit. We use a rotating coordinate system. The three bodies determine a plane in space and a two-dimensional cartesian coordinate system in this plane. The origin is at the center of mass of the two heavy bodies, the x axis is the line through these two bodies, and the distance between them is taken as the unit of length. Thus if μ is the ratio of the mass of the moon to that of the earth, then the moon and the earth are located at coordinates $(1-\mu, 0)$ and $(-\mu, 0)$ respectively, and the coordinate system moves as the moon revolves about the earth. The third body, the spacecraft, is assumed to have a mass which is negligible compared to the other two, and its position as a function of time is $(x(t), y(t))$. The equations are derived from Newton's law of motion and the inverse square law of gravitation. The first derivatives in the equation come from the rotating coordinate system and from a frictional term, which is assumed to be proportional to velocity with proportionality constant f:

$$\begin{aligned} x'' &= 2y' + x - \frac{\tilde{\mu}(x+\mu)}{r_1^3} - \frac{\mu(x-\tilde{\mu})}{r_2^3} - fx' \\ y'' &= -2x' + y - \frac{\tilde{\mu}y}{r_1^3} - \frac{\mu y}{r_2^3} - fy', \end{aligned}$$

with

$$\mu = \frac{1}{82.45},\ \tilde{\mu} = 1-\mu,\ r_1^2 = (x+\mu)^2 + y^2,\ r_2^2 = (x-\tilde{\mu})^2 + y^2.$$

It is known that the initial conditions

$$x(0) = 1.2,\ x'(0) = 0,\ y(0) = 0,\ y'(0) = -1.04935750983032$$

lead to a solution which is periodic with period $T = 6.1921693313196$, when $f = 0$. This means that the spacecraft starts on the far side of the moon with an altitude of about 0.2 times the earth-moon distance and a certain initial velocity. The resulting orbit brings the spacecraft in close to the earth, out in a big loop on the opposite side of the earth from the moon, back in close to the earth again, and finally back to its original position on the far side of the moon with velocity the same as the initial velocity. (See Case Study 6, p.244, *Shampine,Allen & Pruess.*)

(a) Use **Rke** to compute the solution with the given initial conditions (i.e. try to reproduce Figure 6.3 on p.245). Verify that the solution is periodic with the given period. How close does the spacecraft come to the *surface* of the earth ? In the equation, distances are measured from the centers of the earth and moon. Assume that the moon is 238,000 miles from earth and that the earth is a sphere with radius 4000 miles. Note that the origin of the coordinate system is within this sphere but not at its center. (The MATLAB function **min** is useful here.)

(b) When $f = 1$ with the same initial conditions as in (a), integrate from $0 \leq t \leq 5$. In this case the spacecraft is captured by the earth and eventually crashes. When does it crash ?

(c) When $f = 0.1$ repeat the computation of (b). What is happening now ? Perform the integration for a longer time, say to $t = 8$.