AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #5 Due July 16, 2004

1.
   (a) Implement the bisection method in MATLAB to find the smallest positive root of
   $$e^{-x} = \cos x \tag{1}$$
   (b) Solve (1) using the secant method. (Use either a calculator or MATLAB.)

2. Write a MATLAB function Newton(f, df, x, tol) to implement Newton's method. You need to supply functions $f(x)$ and $df(x)(f'(x))$. The input x is the initial guess and tol is the desired accuracy which should be attained when $|x_{i+1} - x_i| < tol$. You should limit the number of iterations and report a failure to converge. Use the **error** function.
   (a) Try your function to solve equation (1). Print out the iterates and the function values.
   (b) Use your function to find the first ten <u>positive</u> solutions of
   $$x = \tan x.$$
   (Zero is not a positive number.) Note: The careful selection of x is critical.
   (c) Try the function on the double root $x = 2$ of
   $$x^3 - x^2 - 8x + 12 = 0.$$
   Use $x = 3$ and $tol = 10^{-6}$. What is the rate of convergence ?

3. Let
   $$g(x) = 1 + 0.3\sin(x).$$
   (a) Show that the equation $g(x) = x$ has exactly one solution, α.
   (b) Find an interval $[a, b]$ such that $g([a, b]) \subset [a, b]$ and $|g'(x)| \le \lambda < 1$ for all $x \in [a, b]$ so that the contraction mapping theorm applies.
   (c) Find α using fixed point iterations.

4. Apply the Aitken extrapolation scheme;
   $$y = g(x), z = g(y), x = z - \frac{(y-z)^2}{((z-y)-(y-x))}$$
   to find the fixed point of $g(x) = 2e^{-x}$ starting with $x_0 = 0.8$. How does the speed of convergence compare with that of fixed point iterations ?

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5. Which of the following iterations will converge to the indicated fixed point α (provided x_0 is sufficiently close to α)? If it does converge, give the order of convergence; for linear convergence, give the rate of linear convergence.

   (a) $$x_{n+1} = -16 + 6x_n + \frac{12}{x_n} \qquad \alpha = 2$$

(b)
$$x_{n+1} = \frac{2}{3}x_n + \frac{1}{x_n^2} \qquad \alpha = 3^{1/3}$$

(c)
$$x_{n+1} = \frac{12}{1+x_n} \qquad \alpha = 3$$

6. Ex 4.16, p.23 *Numerical Computing with MATLAB.*

7. The study of neutron transport in a rod leads to a transcendental equation that has roots related to the critical length. For a rod of length l the equation is

$$\cot(lx) = \frac{x^2-1}{2x}.$$

Graph the two fuctions $\cot(lx)$ and $(x^2-1)/2x$ to get an idea of where they intersect to yield roots. For $l = 1$, determine the three smallest positive roots. (Use **fzero** or **fzerotx**.)

8. To solve the nonlinear two-point boundary value problem

$$y'' = e^y - 1, \;\; y(0) = 0, y(1) = 3$$

using standard initial value codes it is necessary to find the missing initial condition $y'(0)$. Observing that $y'' = \exp(y) - 1$ can be written in the form

$$\frac{d}{dx}\Big[\frac{(y')^2}{2} - e^y + y\Big] = 0$$

we can integrate to obtain

$$\frac{(y')^2}{2} - e^y = y = c, \text{a constant}$$

Since $y(0) = 0$, this says $y'(0) = \sqrt{2c+2}$. Solving for $y'(x)$ (by separation of variables) yields

$$\sqrt{2}x = \int_0^y \frac{dy}{\sqrt{c + e^y - y}},$$

which, when evaluated at $x = 1$, becomes

$$\sqrt{2} = \int_0^3 \frac{dy}{\sqrt{c + e^y - y}}$$

Use **fzero** and **quad** to find c and then $y'(0)$

9. Solve the system

$$x^2 + xy^3 = 9 \qquad 3x^2y - y^3 = 4$$

using Newton's method for nonlinear systems. Use each of the initial guesses $(x_0, y_0) = (1.2, 2.5), (-2, 2.5), (-1.2, -2.5), (2, -2.5)$. Observe which root to which the method converges and the number of iterates required. Write a MATLAB function with the initial guess as input. Be sure to take advantage of the fact that MATLAB works with vectors.

10. Consider the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{pmatrix} 4 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ -1 & 0 & -1 & 4 \end{pmatrix}$$

and $\mathbf{b} = (-2, 4, 6, 12)'$. Solve the system using

(a) The Cholesky factorization of A (MATLAB: CHOL)

(b) Jacobi iteration.

(c) Gauss-Seidel iteration. (The MATLAB command TRIL might be useful.)